Homework 1 (due 3/12)

1. This question concerns the parabola $y^2 = 4ax (a > 0)$ with parametric equations $x = at^2, y = 2at$ and focus F. Let P and Q be points on the parabola with parameters t_1 and t_2, respectively.

   (a) If PQ subtends a right angle at the vertex O of the parabola, prove that $t_1 \cdot t_2 = -4$.

   (b) If $t_1 = 2$ and PQ is perpendicular to OP, prove that $t_2 = -4$.

2. This question concerns the rectangular hyperbola $xy = c^2 (c > 0)$ with parametric equations $x = ct, y = c/t$. Let P and Q be points on the hyperbola with parameters $t_1 (t_1 > 0)$ and $t_2 (t_2 > 0)$, respectively.

   (a) Determine the equation of the chord PQ.

   (b) Determine the coordinates of the point N where PQ meets the x-axis.

   (c) Determine the midpoint M of PQ.

   (d) Prove that $OM = MN$, where O is the origin.

3. Classify the conics in $\mathbb{R}^2$ with the following equations. Determine the center/vertex and axis of each.

   (a) $x^2 - 3xy + y^2 + 4x - 5y + 2 = 0$

   (b) $x^2 + 3xy + 4y^2 - 7 = 0$

   (c) $x^2 + 2xy + y^2 - 7x + 3 = 0$