Homework 2 (due 3/19)

1. The Euclidean transformations t_1 and t_2 are given by

$$t_1(\mathbf{x}) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

   and

$$t_2(\mathbf{x}) = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

   Determine the composite $t_2^{-1} \circ t_1$.

2. Write down an example (if one exists) of each type of transformation $t : \mathbb{R}^2 \to \mathbb{R}^2$ described below. In each case, justify your answer.

   (a) An affine transformation t which is not a Euclidean transformation

   (b) A Euclidean transformation t which is not an affine transformation

   (c) A transformation t which is both Euclidean and affine

   (d) A transformation t which is one-to-one, but is neither Euclidean nor affine

3. Determine the affine transformation which maps the points $(1, -1), (5, -4)$ and $(-2, 1)$ to the points $(1, 1), (4, 0)$ and $(0, 2)$, respectively.

4. Determine the equation of the image of the parabola P with equation $y = x^2$ under the affine transformation $t : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$t(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \mathbf{x}.$$

   Show the image of the vertex of P is not the vertex of $t(P)$. (This proves that the property of "being a vertex of a parabola" is not an affine property.)