Notes for Geometry
Recognizing Conics - Exercises

Problems are taken from *Geometry, by David A. Brannan, Matthew F. Esplen and Jeremy J. Gray, 2nd edition*

1. Let the Euclidean transformations t_1 and t_2 of $\mathbb{R}^2$ be given by

$$t_1(\mathbf{x}) = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

   and

$$t_2(\mathbf{x}) = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

   Determine $t_1 \circ t_2$ and $t_2 \circ t_1$.

2. Prove that if t_1 is a Euclidean transformation of $\mathbb{R}^2$ given by

$$t_1(\mathbf{x}) = U\mathbf{x} + a \quad (\mathbf{x} \in \mathbb{R}^2),$$

   then

   (a) the transformation of $\mathbb{R}^2$ given by

$$t_2(\mathbf{x}) = U^{-1}\mathbf{x} - U^{-1}a \quad (\mathbf{x} \in \mathbb{R}^2)$$

   is also a Euclidean transformation;

   (b) the transformation t_2 is the inverse of t_1.

3. Determine the inverse of the Euclidean transformation given by

$$t(\mathbf{x}) = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

4. Which of the following sets consists of figures that are Euclidean-congruent to each other?

   (a) The set of all ellipses

   (b) The set of all line segments of length 1

   (c) The set of all triangles

   (d) The set of all squares that have sides of length 2