Homework 1 (due 3/2)

1. Let P be a point $(\sec t, \frac{1}{\sqrt{2}} \tan t)$, where $t \in (-\pi/2, \pi/2) \cup (\pi/2, 3\pi/2)$, on the hyperbola E with equation $x^2 - 2y^2 = 1$.

   (a) Determine the foci F and F' of E.

   (b) Determine the slopes of FP and $F'P$, when these lines are not parallel to the y-axis.

   (c) Determine the point P in the first quadrant on E for which FP is perpendicular to $F'P$.

2. This question concerns the parabola $y^2 = 4ax (a > 0)$ with parametric equations $x = at^2, y = 2at$ and focus F. Let P and Q be points on the parabola with parameters t_1 and t_2, respectively.

   (a) If PQ subtends a right angle at the vertex O of the parabola, prove that $t_1 \cdot t_2 = -4$.

   (b) If $t_1 = 2$ and PQ is perpendicular to OP, prove that $t_2 = -4$.

3. This question concerns the rectangular hyperbola $xy = c^2 (c > 0)$ with parametric equations $x = ct, y = c/t$. Let P and Q be points on the hyperbola with parameters $t_1 (t_1 > 0)$ and $t_2 (t_2 > 0)$, respectively.

   (a) Determine the equation of the chord PQ.

   (b) Determine the coordinates of the point N where PQ meets the x-axis.

   (c) Determine the midpoint M of PQ.

   (d) Prove that $OM = MN$, where O is the origin.

4. Let E be the conic in $\mathbb{R}^2$ with the equation

$$5x^2 + 4xy + 5y^2 + 20x + 8y - 1 = 0.$$

   Use the methods of linear algebra to answer the following questions.

   (a) To classify the conic E.

   (b) Write the equation in standard form.

   (c) Determine its center/vertex and axis.

   (d) Sketch the graph.