Homework 9 (due 6/1)

1. Exercise 2.4.6.

2. Exercise 2.4.13(a), (b).

3. Let T be a torus parametrized by

$$\mathbf{x}(u, v) = ((a + b\cos u)\cos v, (a + b\cos u)\sin v, b\sin u),\ a > b.$$

Prove that

(a) If a geodesic is tangent to the parallel $u = \frac{\pi}{2}$, then it is entirely contained in the region of T given by $-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$.

(b) A geodesic that intersects the parallel $u = 0$ under an angle $\phi\,(0 < \phi < \frac{\pi}{2})$ also intersects the parallel $u = \pi$ if $\cos\phi < \frac{a-b}{a+b}$.

Hint: Use Clairaut's relation. You might also need to use the result of Exercise 2.4.6.