NAME:________________ ID NO.:______________________ CLASS: ___________

1. (10 pts) Determine the circle of the line $x^2 + y^2 = 1$ under the affine transformation
$$t(\mathbf{x}) = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -1 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -\frac{3}{2} \\ 4 \end{pmatrix} \qquad (\mathbf{x} \in \mathbb{R}^2).$$

   Solution. $10x^2 + 6xy + y^2 + 6x + y + 2 = 0$. □

2. (10 pts) Determine the Point of intersection of the Line in $\mathbb{R}P^2$ with equations $x - y - z = 0$ and $x + 5y + 2z = 0$.

   Solution. $[1, -1, 2]$ or $[-1, 1, -2]$. □

3. Let $A = [1, -1, -1]$, $B = [1, 3, -2]$ and $C = [3, 5, -5]$ be three Points in $\mathbb{R}P^2$ in homogeneous coordinates.

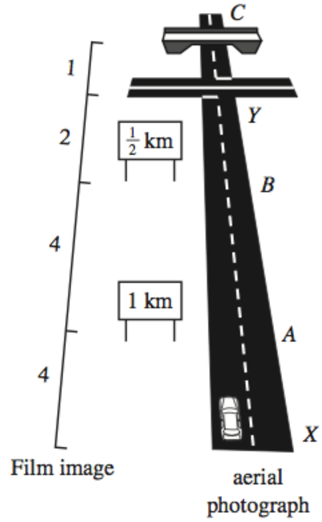
   (a) (5 pts) Show that A, B and C are collinear.

   Solution. $\det \begin{pmatrix} 1 & 1 & 3 \\ -1 & 3 & 5 \\ -1 & -2 & -5 \end{pmatrix} = 0.$ □

   (b) (10 pts) Find the Point $D = [a, b, c]$ on the Line through the Points A and B such that the cross-ratio $(ABCD) = -4$.

   Solution. $[1, -5, 0]$. □

4. (15 pts) The diagram represents an aerial photograph of a straight road on flat ground. At A there is a sign "Junction 1 km", at B a sign "Junction $\frac{1}{2}$ km", and J is the road junction. Also, a police patrol car is at X, and a bridge is at C. The distances marked on the left of the diagram are measured in cm from the photograph. Calculate the actual distances (in km) of the patrol car and the bridge from the junction.



Solution. The actual distance of the patrol car from the junction is $5/4$ km and the actual distance of the bridge from the junction is $2/5$ km. □

5. (a) (5 pts) Use the determinant of a matrix to classify the non-degenerate conic
$$3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$$
in $\mathbb{R}^2$.

Solution. hyperbola □

(b) (5 pts) Find the equation for the projective figure in $\mathbb{RP}^2$ which corresponds to the conic $\{(x, y, z) : 3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0, z = 1\}$ in the standard embedding plane.

Solution. $3x^2 - 10xy + 3y^2 + 14xz - 2yz + 3z^2 = 0$. □

(c) (5 pts) Which ideal Points should be associated with this projective figure?

Solution. $[1, 3, 0], [3, 1, 0]$. □

6. Let E be the conic in $\mathbb{R}^2$ with the equation
$$x^2 - 4xy + 4y^2 - 6x - 8y + 5 = 0.$$
Use the methods of linear algebra to answer the following questions.

(a) (5 pts) To classify the conic E.

Solution. parabola $\square$

(b) (10 pts) Write the equation in standard form.

Solution. $(y-\frac{1}{\sqrt{5}})^2 = \frac{4}{\sqrt{5}}x-\frac{4}{5}$ or $(x-\frac{1}{\sqrt{5}})^2 = \frac{4}{\sqrt{5}}y-\frac{4}{5}$ or $(y+\frac{1}{\sqrt{5}})^2 = \frac{4}{\sqrt{5}}x-\frac{4}{5}$ or $(x+\frac{1}{\sqrt{5}})^2 = \frac{4}{\sqrt{5}}y-\frac{4}{5}$. $\square$

(c) (5 pts) Determine its center/vertex and axis in original (x, y) coordinates.

Solution. vertex: $(1/5, 3/5)$, axix: $x - 2y + 1 = 0$. $\square$

7. Determine the affine transformation which maps the points

(a) (5 pts) $(1, -1), (2, -2)$ and $(3, -4)$ to $(0, 0), (1, 0)$ and $(0, 1)$, respectively.

Solution. $\begin{pmatrix} 3 & 2 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ $\square$

(b) (5 pts) $(0, 0), (1, 0)$ and $(0, 1)$ to $(8, 13), (3, 4)$ and $(0, -1)$, respectively.

Solution. $\begin{pmatrix} -5 & -8 \\ -9 & -14 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8 \\ 13 \end{pmatrix}$ $\square$

(c) (5 pts) $(1, -1), (2, -2)$ and $(3, -4)$ to $(8, 13), (3, 4)$ and $(0, -1)$, respectively.

Solution. $\begin{pmatrix} -7 & -2 \\ -13 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 13 \\ 22 \end{pmatrix}$ $\square$