1.6.29(a):

Proof. $\dim(W_1 \cap W_2) \leq \dim(V)$
$\Rightarrow W_1 \cap W_2$ has a finite basis $\beta = \{u_1, u_2, \cdots, u_k\}$.
We can extend β to a basis $\beta_1 = \{u_1, u_2, \cdots, u_k, v_1, v_2, \cdots, v_m\}$ for W_1 and to a basis $\beta_2 = \{u_1, u_2, \cdots, u_k, k_1, k_2, \cdots, k_p\}$ for W_2.
Let $\alpha = \{u_1, u_2, \cdots, u_k, v_1, v_2, \cdots, v_m, w_1, w_2, \cdots, w_p\}$.
We claim that α is a basis for $W_1 + W_2$.
To prove the claim, we need to check that

1. α is linearly independent.

Let $a_1u_1 + \cdots + a_ku_k + b_1v_1 + \cdots + b_mv_m + c_1w_1 + \cdots + c_pw_p = 0$, for some scalars $a_1, \cdots, a_k, b_1, \cdots, b_m, c_1, \cdots, c_p$.
Then $(-b_1)v_1 + \cdots + (-b_m)v_m = a_1u_1 + \cdots + a_ku_k + c_1w_1 + \cdots + c_pw_p \in W_1 \cap W_2$.
Since β is a basis for $W_1 \cap W_2$, we have $(-b_1)v_1 + \cdots + (-b_m)v_m = d_1u_1 + \cdots + d_ku_k$ for some scalars $d_1, \cdots, d_k$.
$\Rightarrow d_1u_1 + \cdots + d_ku_k + b_1v_1 + \cdots + b_mv_m = 0$
$\Rightarrow d_1 = \cdots = d_k = b_1 = \cdots = b_m = 0$, since β_1 is a basis for W_1.
$\Rightarrow a_1u_1 + \cdots + a_ku_k + c_1w_1 + \cdots + c_pw_p = 0$
$\Rightarrow a_1 = \cdots = a_k = c_1 = \cdots = c_p = 0$, since β_2 is a basis for W_2.
Hence α is linearly independent.

2. $W_1 + W_2 = \text{span}(\alpha)$.

Let $u = v + w \in W_1 + W_2$, where $v \in W_1$ and $w \in W_2$, be any vector in $W_1 + W_2$.
Since β_1 is a basis for W_1 and β_2 is a basis for W_2, we can find some scalars $x_1, \cdots, x_k, y_1, \cdots, y_m, z_1, \cdots, z_k, t_1, \cdots, t_p$ such that

$$\begin{aligned} u &= (x_1u_1 + \cdots + x_ku_k + y_1v_1 + \cdots y_mv_m) + (z_1u_1 + \cdots + z_ku_k + t_1w_1 + \cdots + t_pw_p) \\ &= \big((x_1 + z_1)u_1 + \cdots + (x_k + z_k)u_k + y_1v_1 + \cdots y_mv_m + t_1w_1 + \cdots + t_pw_p\big) \end{aligned}$$

That is, $W_1 + W_2 \subseteq \text{span}(\alpha)$.
It is easy to see that $\text{span}(\alpha) \subseteq W_1 + W_2$.
Hence $W_1 + W_2 = \text{span}(\alpha)$.
Therefore, α is a basis for $W_1 + W_2$.

Finally, we have

$\dim(W_1+W_2) = k+m+p = (k+m)+(k+p)-k = \dim(W_1)+\dim(W_2)-\dim(W_1 \cap W_2)$.

□