

2.5.9:

Proof. 1. (Reflexivity) $A = I^{-1}AI$, so A is similar to A .

2. (Symmetry) If A is similar to B , i.e. there exists a matrix Q such that $A = Q^{-1}BQ$, then $B = (Q^{-1})^{-1}AQ^{-1}$, i.e. B is similar to A .

3. (Transitivity) If A is similar to B , i.e. there exists a matrix Q such that $A = Q^{-1}BQ$, and B is similar to C , i.e. there exists a matrix P such that $B = P^{-1}CP$, then $A = (PQ)^{-1}C(PQ)$, i.e. A is similar to C . Hence "is similar to" is an equivalence relation on $M_{n \times n}(F)$. \square