4.2.23: Let $A \in M_{n\times n}(F)$ be an upper triangular matrix. We want to show that

$$\det(A) = \Pi_{i=1}^{n} A_{ii}.$$

Proof. We proceed by induction on n.
For $n = 1$, obviously $\det(A) = A_{11}$.
Assume that the statement holds for $n - 1$.
Since A is an upper triangular matrix, we know that $A_{ni} = 0$, for $i = 1, 2, \cdots, n-1$.
Then Cofactor expansion along the n-th row of A gives

$$\det(A) = A_{nn} \det\left(\widetilde{A}_{nn}\right).$$

Since $\widetilde{A}_{nn}$ is the matrix obtained from A by deleting the n-th row and the n-th column, and hence is a $(n-1) \times (n-1)$ matrix.
By induction hypothesis,

$$\det\left(\widetilde{A}_{nn}\right) = \Pi_{i=1}^{n-1} A_{ii}.$$

Therefore

$$\det(A) = A_{nn} \det\left(\widetilde{A}_{nn}\right) = \Pi_{i=1}^{n} A_{ii}.$$

□