**1.4.16:**

*Proof.* Assume that $v \in \text{span}(S)$.
Assume that for some scalars $a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_n$ and $v_1, v_2, \cdots, v_n \in S$, we have

$$v = a_1v_1 + a_2v_2 + \cdots + a_nv_n = b_1v_1 + b_2v_2 + \cdots + b_nv_n.$$

Then $0 = v - v = (a_1 - b_1)v_1 + (a_2 - b_2)v_2 + \cdots + (a_n - b_n)v_n$.
By assumption, we have $a_1 - b_1 = a_2 - b_2 = \cdots = a_n - b_n = 0$.
That is $a_1 = b_1, a_2 = b_2, \cdots, a_n = b_n$.
Hence every vector in the span of $S$ can be uniquely writen as a linear combination of vectors of $S$. □