2.2.13:

Proof. Suppose that $\{T, U\}$ is linearly dependent. Then there exists some nonzero scalar $cT = U$. Since T is a nonzero transformation from V to W, there exists some vector $u \in V$ and some nonzero vector $v \in W$ such that $T(u) = v \neq 0$. Then $U(u) = cv$. But we also have $v = \frac{1}{c}(cv) = \frac{1}{c}U(u) = U(\frac{1}{c}u) \in R(U)$. This implies that $0 \neq v \in R(T) \cap R(U)$ which contradicts our assumption. Hence $\{T, U\}$ is linearly independent. □

2.2.15(c):

Proof. Since $V_1 \subseteq V_1 + V_2$, by (b), we have $(V_1 + V_2)^0 \subseteq V_1^0$. Similarly, we have $(V_1 + V_2)^0 \subseteq V_2^0$. Hence $(V_1 + V_2)^0 \subseteq V_1^0 \cap V_2^0$.
Now assume that $T \in V_1^0 \cap V_2^0$. Then for x in V_1 or V_2, we have $T(x) = 0$. Let $v = v_1 + v_2 \in V_1 + V_2$, where $v_1 \in V_1$ and $v_2 \in V_2$, then

$$T(v) = T(v_1 + v_2) = T(v_1) + T(v_2) = 0 + 0 = 0.$$

This implies $T \in (V_1 + V_2)^0$, then $V_1^0 \cap V_2^0 \subseteq (V_1 + V_2)^0$. Therefore $(V_1 + V_2)^0 = V_1^0 \cap V_2^0$. □

2.2.16:

Proof. Assume that $\dim V = \dim W = n$. Let $\{v_1, \cdots, v_k\}$ be a basis for $N(T)$. Then by Replacement Theorem, we can extend it to a basis
$\beta = \{v_1, \cdots, v_k, v_{k+1}, \cdots, v_n\}$ for V. Let $T(v_i) = w_i$, for $i = k+1, \cdots, n$.
Claim: $\{w_{k+1}, \cdots, w_n\}$ is linearly independent.
$\cdots\cdots$
$\cdots\cdots$
Since $\{w_{k+1}, \cdots, w_n\}$ is linearly independent, again by Replacement Theorem, we can extend it to a basis $\gamma = \{w_1, \cdots, w_k, w_{k+1}, \cdots, w_n\}$ for W.
$\cdots\cdots$
$\cdots\cdots$
We find that $[T]_\beta^\gamma$ is the diagonal matrix $\begin{pmatrix} 0 & 0 \\ 0 & I_{n-k} \end{pmatrix}$, where I_{n-k} is the $(n-k) \times (n-k)$ identity matrix. □