2.4.9:

Proof. If AB is invertible, then L_{AB} is invertible. So $L_{AB} = L_A L_B$ is injective and surjective. By 2.3.12(a)(b), we have L_B is injective and L_A is surjective. Since L_A and L_B are linear transformations from $\mathbb{F}^n$ to $\mathbb{F}^n$. By Theorem 2.5, L_B is surjective and L_A is injective. So both L_A and L_B are invertible. Hence A and B are invertible. □

2.4.13:

Proof. 1. (Reflexivity) $I_V : V \to V$ is an isomorphism.
2. (Symmetry) If $T : V \to W$ is an isomorphism, then $T^{-1} : W \to V$ is an isomorphism also.
3. (Transitivity) If $T : V \to W$ is an isomorphism and $U : W \to Z$ is an isomorphism, then $UT : V \to Z$ is an isomorphism also. Hence $\sim$ is an equivalence relation on the class of vector spaces over F. □

2.4.15:

Proof. ($\Rightarrow$) T is an isomorphism. $\Rightarrow$ T is invertible. $\Rightarrow$ T is injective and surjective. Since T is injective, by 2.1.14 (a), we know that $T(\beta)$ is linearly independent. Since T is surjective, we know that $\text{span}(T(\beta)) = R(T) = W$. Hence $T(\beta)$ is a basis for W.
($\Leftarrow$) Since $T(\beta)$ is a basis, we know $\text{span}(T(\beta)) = R(T) = W$. This implies that T is surjective and $\dim R(T) = \dim W = n$. By assumption, we have $\dim V = \dim W = n$. By Dimension Theorem, we have $n = \dim V = \dim R(T) + \dim N(T)$. This implies that $\dim N(T) = 0$, i.e. $N(T) = \{0\}$. So T is injective. Therefore, we show that T is an isomorphism. □