3.1.6:

Proof. If B can be obtained from A by an elementary row [column] operation, then $B = EA$ $[B = AE]$. So we have $B^t = (EA)^t = A^tE^t$. $[B^t = (AE)^t = E^tA^t]$ and this means that B^t can be obtained by A^t by elementary column [row] operation with corresponding elementary matrix E^t. □

3.1.8:

Proof. If Q can be obtained from P by an elementary row operation, then we can write $Q = EP$. So we have $P = E^{-1}Q$. By Theorem 3.2, E^{-1} is an elementary matrix of the same type as E is. Hence P can be obtained from Q by an elementary row operation of the same type. □

3.1.9:

Proof. An elementary row operation of type 1 can be obtained by a succession of the following steps:

(1) adding $(-1)\times$ the i-th row to the j-row (e.r.o. of type 3);
(2) adding the j-th row to the i-th row (e.r.o. of type 3);
(3) adding $(-1)\times$ the i-th row to the j-row (e.r.o. of type 3);
(4) multiplying the i-th row by (-1) (e.r.o. of type 2).

□

3.1.12:

Proof. We will prove the assertion by induction on the number of rows m.
If $m = 1$, then there is nothing to prove.
Suppose that the assertion holds for $m - 1$.
Let j be the index of the first column of A that has nonzero entry so that $A_{ij'} = 0$ for $i = 1, \cdots, m, j' < j$. By a sequence of elementary row operations of type 1, we may assume that $A_{1j} \neq 0$. By adding $-\frac{A_{ij}}{A_{1j}}$ times the first row to the i-th row for $i = 2, \cdots, m$, we obtain a new matrix A'. Let B be the $(m-1) \times n$-matrix by deleting the first row of A'. Then by induction assumption we can make B an upper triangular matrix by a sequence of elementary row operations of types 1 and 3. Hence, we transforms A into an upper triangular matrix by a sequence of elementary row operations of types 1 and 3. □