Linear Algebra — Quiz 6

Name:____________ Id No.:____________________ Class: __________

Problem 1: (6 points) Express the invertible matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ as a product of elementary matrices.

Solution. For example,

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\square$

Problem 2: For each of the following linear transformations T, determine whether T is invertible, and compute T^{-1} if it exists.

(1) (6 points) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(a_1, a_2, a_3) = (\, a_1 + 2a_2 + a_3, -a_1 + a_2 + 2a_3, a_1 + a_3 \,).$$

Solution.

$$[T]_\alpha^\beta = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}, \quad [T^{-1}]_\beta^\alpha = \begin{pmatrix} \frac{1}{6} & -\frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix}$$

$$T(a, b, c) = \left(\frac{1}{6}a - \frac{1}{3}b + \frac{1}{2}c, \frac{1}{2}a - \frac{1}{2}c, -\frac{1}{6}a + \frac{1}{3}b + \frac{1}{2}c \right).$$

$\square$

(2) (6 points) $T : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}^4$ defined by

$$T(A) = \big(\operatorname{tr}(A), \operatorname{tr}(A^t), \operatorname{tr}(EA), \operatorname{tr}(AE) \big),$$

where $E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Solution. T is not invertible, since T is not one-to-one, ex. $T(A) = T(A^t)$.

$\square$