LINEAR ALGEBRA QUIZ 7

NAME:____________ ID NO.:____________________ CLASS: __________

Problem 1: Let V denote the set of all solutions to the system of linear equations

$$x_1 - x_2 + 2x_4 - 3x_5 + x_6 = 0$$
$$2x_1 - x_2 - x_3 + 3x_4 - 4x_5 + 4x_6 = 0.$$

(1) (8 points) Find the dimension of and a basis for V.

Solution. $\dim V = 4$ and

$$\beta = \{(1,1,1,0,0,0), (-1,1,0,1,0,0), (1,-2,0,0,1,0), (-3,-2,0,0,0,1)\}$$

is a basis for V. □

(2) (2 points) Show that $S = \{(1,0,1,1,1,0), (0,2,1,1,0,0)\}$ is a linearly independent subset of V.

Solution. Let $a(1,0,1,1,1,0) + b(0,2,1,1,0,0) = (0,0,0,0,0,0)$, then $a = b = 0$. Hence S is a linearly independent subset of V. □

(3) (8 points) Extend S to a basis for V.

Solution. $\{(1,0,1,1,1,0), (0,2,1,1,0,0), (1,1,1,0,0,0), (-3,-2,0,0,0,1)\}$ is a basis for V. □