5.3.4. If A is diagonalizable, then, by Theorem 5.1, there exists an invertible matrix Q such that

$$Q^{-1}AQ = D = \begin{pmatrix} \lambda_1 & 0\cdots & 0 & \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix},$$

where $\lambda_1, \cdots, \lambda_n$ are eigenvalues of A. By Theorem 5.13 if $\lim_{m\to\infty} D^m$ exists, then $\lambda_1, \cdots, \lambda_n \in S = \{\lambda \in \mathbb{C} : |\lambda| < 1 \text{ or } \lambda = 1\}$.

(1) If $\lambda_1 = \cdots = \lambda_n = 1$, then $L = I_n$.

(2) If some $\lambda_i \neq 1$, then $|\lambda_i| < 1$ and $\lim_{m\to\infty} \lambda_i^m = 0$. This implies that $\operatorname{rank}(L) < n$.

□

5.3.21.

$$\begin{aligned} e^A &= \lim_{m\to\infty} B_m \\ &= \lim_{m\to\infty} \left(I + A + \frac{A^2}{2!} + \cdots + \frac{A^m}{m!} \right) \\ &= \lim_{m\to\infty} \left(I + PDP^{-1} + \frac{(PDP^{-1})^2}{2!} + \cdots + \frac{(PDP^{-1})^m}{m!} \right) \\ &= \lim_{m\to\infty} P \left(I + D + \frac{D^2}{2!} + \cdots + \frac{D^m}{m!} \right) P^{-1} \\ &= P \lim_{m\to\infty} \left(I + D + \frac{D^2}{2!} + \cdots + \frac{D^m}{m!} \right) P^{-1} \\ &= Pe^D P^{-1}. \end{aligned}$$

□