Linear Algebra II Quiz 7

Name:_____ Id No.:_____ Class: _____

Problem 1: Let $A = \begin{pmatrix} 2 & 0 & 0 & -1 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 2 \end{pmatrix}$.

(1) (5 points) Find a Jordan canonical form J of A.

Solution. $J = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$. □

(2) (5 points) Find a matrix Q such that $Q^{-1}AQ = J$.

Solution. $Q = \begin{pmatrix} 0 & -1 & 0 & 1 \\ 3 & 5 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$. □

Problem 2: Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{pmatrix}$.

(1) (5 points) Find a Jordan canonical form J of A.

Solution. $J = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. □

(2) (5 points) Find a matrix Q such that the first row of Q is $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$ and $Q^{-1}AQ = J$.

Solution. $Q = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$. □