There are ten problems in this test, and each problem has the value of ten points.

1. Solve the first order equation for $y = y(x)$:

$$(e^y + 2xy^2) + (xe^y + 2x^2y)\frac{dy}{dx} = 0.$$

2. Find a fundamental set of real-valued solutions for the homogeneous equation:

$$y'' - 6y' + 25y = 0.$$

3. Solve the nonhomogeneous equation for $y = y(x)$:

$$y'' - 3y' + 2y = e^{3x}.$$

4. Solve the nonhomogeneous equation for $y = y(x)$:

$$y'' + y = \sec(x).$$

5. Find a fundamental set of solutions for the first order, linear system:

$$\frac{du(t)}{dt} = \begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix} u(t).$$

6. Solve the first order, linear, nonhomogeneous system for $u = u(t)$:

$$\frac{du(t)}{dt} = \begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix} u(t) + \begin{pmatrix} 0 \\ t \end{pmatrix}.$$

7. Find a fundamental set of real-valued solutions for the system:

$$\frac{du(t)}{dt} = \begin{pmatrix} -3 & -2 \\ 9 & 5 \end{pmatrix} u(t).$$

8. Consider the boundary value problem:

$$y'' + 4y = f(x), \quad 0 < x < 1;$$
$$y(0) = 0 = y(1);$$

where $f(x)$ is a continuous function on $[0, 1]$. Find the corresponding Green's function $G(x.\xi)$, and express the solution $y = y(x)$ in terms of $G(x, \xi)$.

9. Find the eigenvalues λ's and the corresponding eigenfunctions y's for the boundary value problem:

$$y'' + \lambda y = 0, \quad 0 < x < b;$$
$$y(0) = 0 = y(b);$$

where $b > 0$ is a constant.

10. Use the technique of Laplace transform to solve the initial value problem:

$$y'' + 4y = t, \quad 0 < t < \infty;$$
$$y(0) = 0 = y'(0).$$