國立中央大學九十四學年度數學系博士班招生筆試試題卷

歸檔

科目： 圖論

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1. (20%) If three distinct integers are chosen, there will always be at least two, say a and b, such that $a^3b - ab^3$ is a multiple of 10.

2. (20%) The hypercube Q_k is the simple graph whose vertices are the ordered k-tuples with entries in {0,1}, two vertices being joined if and only if they differ in exactly one position. Prove that Q_k is k-connected.

3. (20%) Let $\tau(G)$ denote the number of spanning trees of a graph G. Find the following values and prove your results.

   (a) $\tau(K_n)$.

   (b) $\tau(K_n - e)$.

4. (20%) The complement $\overline{G}$ of a simple graph G is the simple graph with vertex set $V(G)$ defined by $uv \in E(\overline{G})$ if and only if $uv \notin E(G)$. Let G be an n-vertices graph and $\chi(G)$ denote the chromatic number of G. Prove the following inequalities.

   (a) $2\sqrt{n} \leq \chi(G) + \chi(\overline{G}) \leq n + 1$.

   (b) $n \leq \chi(G) \cdot \chi(\overline{G}) \leq (\frac{n+1}{2})^2$.

5. (20%) If C is a cycle, and e is an edge connecting two nonadjacent vertices of C, then we call e a *chord* of C. Prove that if G is simple with $n \geq 4$ vertices and $2n - 3$ edges, then G contains a cycle with a chord.