Algebra 2012-Fall

In this test let Q and C denote the fields of rational numbers and complex numbers, respectively.

1. Let E be an extension field of a field F and let $\alpha \in E$ be transcendental over F. Find the algebraic closure of F in $F(\alpha)$. (16 %)

2. Let A be the ring of all algebraic integers in the field of complex numbers C. Prove or disprove that A is a Dedekind domain. (20 %)

3. Give (with a proof) a unique factorization domain, neither a field nor a local ring, with only a finite number of non-associated prime elements. (16 %)

4. Let Q^a be the subfield of C consisting of all numbers which are algebraic over Q. Prove or disprove that Q^a has the same cardinality as Q. (16 %)

5. Let D be a principal ideal domain and a, b nonzero nonunit elements in D. Suppose $\pi_{ab}(D') \supseteq (D/abD)^{\circ}$. Prove or disprove that $\pi_a(D') \supseteq (D/aD)^{\circ}$, where D' is a subset of D, $\pi_x : D' \to D/xD$ the canonical map with $x \in D$ and $(D/xD)^{\circ}$ the set of all distinct cosets $s + xD$ with s coprime to x. (16 %)

6. Prove or disprove that the field Q of rational numbers is not a projective Z-module, where Z denotes the ring of rational integers. (16 %)