DEPARTMENT OF MATHEMATICS
NATIONAL CENTRAL UNIVERSITY

Ph. D. Qualifying Examination

2002.9.
分析

Fall, 2002

Analysis

1. Suppose $\int_I f(y)dy = 0$ for every subinterval I of $\mathbb{R}$. Show: $f = 0$ a.e. on $\mathbb{R}$.

2. If $f \in L^1(-\infty, \infty)$ then show:

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} f(x) \cos nx \, dx = 0$$

3. Let $F(x, y)$ be continuous and bounded on the unit square $0 < x < 1$, $0 < y < 1$. Show: $f(y) = \overline{\lim}_{x\to 0+} F(x, y)$ is measurable on $(0, 1)$.

4. Suppose $f,\ f_n$ are of bounded variation on $[a, b]$, $n = 1, 2, \cdots$ and

$$V(f_n - f, a, b) \to 0$$

(where $V(f, a, b) =$ the total variation of f on $[a, b]$)
Prove: There is a subsequence $n_k \to \infty$ such that

$$f'_{n_k} \longrightarrow f' \quad \text{a.e.}$$

5. Let $f \in L(\mathbb{R}^n)$. Show

$$\lim_{|h|\to 0} \int_{\mathbb{R}^n} |f(x+h) - f(x)| dx = 0$$

6. Let $f, g \in L^1(\mathbb{R}^1)$. Show:

   (a) $f(x - t)g(t)$ is a measurable function on $\mathbb{R}^2$.

   (b) $\phi(x) = \int_{-\infty}^{\infty} f(x - t)g(t)dt$ is in $L^1(\mathbb{R}^1)$