5. Let λ and μ be two positive Borel measures on $\mathbb{R}^n$ such that λ and μ are finite on compact sets and for every continuous function f on $\mathbb{R}^n$ with compact support,

$$\int_{\mathbb{R}^n} f d\lambda = \int_{\mathbb{R}^n} f d\mu.$$

Show that $\lambda = \mu$.

6. Let $f_n \to f$ on $[0,1]$ in the following sense: for every x in $[0,1]$, if $x_n \to x$, then $f_n(x_n) \to f(x)$. Show that f is continuous if all f_n are continuous.

7. Let $\{G_n\})_n$ be a sequence of non-empty open sets in $[0,1]$ with the Lebesgue measures $m(G_n) \leq 1/2^n$ for $n = 1, 2, \ldots$. Let

$$f(x) = \sum_{n=1}^{\infty} m(G_n \cap [0,1]), \quad 0 \leq x \leq 1.$$

Show that f is continuous, non-decreasing, and that $f'(x) = +\infty$ for all x in $\cap_{n=1}^{\infty} G_n$.

8. Prove or disprove: There exist continuous real-valued functions f and g defined on $[0,1]$ such that $f(x) = g(x)$ for uncountably many points x, but in every interval there exists a point x where $f(x) \neq g(x)$.