

6. (15%) Consider Lotka-Volterra two species competition model

$$\begin{aligned}\frac{dx}{dt} &= \gamma_1 x \left(1 - \frac{x}{K_1}\right) - \alpha xy, \\ \frac{dy}{dt} &= \gamma_2 y \left(1 - \frac{y}{K_2}\right) - \beta xy,\end{aligned}$$

with $\gamma_1 > 0, \gamma_2 > 0, K_1 > 0, K_2 > 0, \alpha > 0$ and $\beta > 0$. Show that there is no periodic orbit in the first quadrant.

7. (10%) Consider linear homogeneous systems $x' = A(t)x$, $-\infty < t < \infty$, where the elements of A are continuous functions on \mathbf{R} and $A(t) = A(t+T)$ for some $T > 0$. Let $\Phi(t)$ be a fundamental matrix for the systems, show that there exists a nonsingular matrix P which is also periodic with period T and a constant matrix D , such that $\Phi(t) = P(t)e^{tD}$.