歸檔

(b) (10 pts) Find the general solution of $x' = Ax$, where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}.$$

3. (a) (10 pts) Consider the following initial value problem

$$\begin{aligned} x' &= 3x\Big(1 - \frac{x}{2}\Big) - \frac{2x}{1+x}y \\ y' &= \Big(\frac{2x}{1+x} - 1\Big)y \\ x(0) &= 1.2,\ y(0) = 1.2 \end{aligned}$$

Please find $\lim_{t\to\infty} x(t)$ and $\lim_{t\to\infty} y(t)$.

(b) (10 pts) Please use the following system

$$\begin{aligned} x' &= -x \\ y' &= y + x^2 \end{aligned}$$

to verify the Stable Manifold Theorem.
Hint. The system can be solved directly.

4. a. (10 pts) Consider the following system

$$\begin{aligned} x' &= y - x\Big(\frac{x^2 - xy + y^2}{x^4 + y^4}\Big) \\ y' &= -x - y\Big(\frac{x^2 - xy + y^2}{x^4 + y^4}\Big) \end{aligned}$$

Discuss the stability of equilibria.

b. (5 pts) Please show the Van der Pol equation

$$x'' = \varepsilon(x^2 - 1)x' + x = 0,\ \varepsilon > 0$$

is equivalent to the Lienard system

$$\begin{aligned} x' + y - \varepsilon\Big(\frac{x^3}{3} - x\Big) \\ y' = -x \end{aligned}$$