5 (25%) ① State the edge version of Menger's Theorem for directed graph.

② State and prove Hall's Theorem (i.e., matching theorem, marriage theorem, SDR theorem).

③ Prove that in a bipartite graph $G$, the maximum number of edges in a matching of $G$ is equal to the minimum number of vertices needed to cover all the edges in $G$

④ Show that every $k$-regular bipartite graph ($k \geq 1$) is 1-factorizable (i.e., contains a perfect matching).

6. (15%) $G$ is a graph of order $n \geq 3$.

① Show that if $\deg x + \deg y \geq n$ for every pair of nonadjacent vertices $x, y$ of $G$, then $G$ contains a Hamiltonian cycle.

② Show that if $\deg x + \deg y \geq n-1$ for every nonadjacent vertices $x, y$ of $G$, then $G$ contains a Hamiltonian path.