中央大學數學系 102(1)

博士班資格考

圖論 2013, 8.

GRAPH THEORY

In the following, graph means finite simple graph.

1. Show that every nontrivial graph G contains a bipartite spanning subgraph H such that $deg_H(v) \geq \frac{1}{2}deg_G(v)$ for all v in $V(G)$. (5 points)

2. Let G be a connected graph of order $n \geq 3$. Suppose that $deg\ x + deg\ y \geq n$ for every pair of nonadjacent vertices x, y in G. Show that G contains a Hamiltonian cycle. (15 points)

3. Let diam G denote the maximum distance between two vertices in a graph G. Show that if diam G ≥ 3, then diam $G^c \leq 3$.(10 points)

4. Let G be a k-regular bipartite graph ($k \geq 1$). Use Hall Theorem to show that G contains a perfect matching. (5 points)

5. Show that every graph is an induced subgraph of a regular graph. (5 points)

6. Let G be a connected graph. Let k(G) and k′(G) denote the connectivity and the edge connectivity, respectively of G. Show that $k(G) \leq k'(G)$. (10 points)

7. For $k, l \in N$, let $r(k, l)$ denote the minimum n such that any graph on n vertices contains either a clique of order k or an independent set of order l. Show that

(1) $r(k+1, l+1) \leq r(k+1, l) + r(k, l+1)$. (10 points)

(2) $r(k, l) \leq \binom{k+l-2}{k-1}$. (5points)

(3) $r(k, k) \geq 2^{k/2}$. (10 points)

8. Let $d_1, d_2, ..., d_n$ be the degree sequence of a graph. Show that
$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} min\{k, d_i\}$ for $1 \leq k \leq n$. (10 points)

9. Let $\chi(G)$ denote the chromatic number of a graph G. Show that
$\chi(G) \geq \frac{|V(G)|^2}{|V(G)|^2 - 2|E(G)|}$. (15 points)