Ph.D. Qualifying Examination

圖論 2013.1

1.(1)(15%) Let G be a graph of order n such that deg x + deg y $\geq$ n for every pair of nonadjacent vertices x and y in G. Show that G contains a Hamiltonian cycle.

(2) (5%) Let G be a graph of order n such that deg x + deg y $\geq$ n-1 for every pair of nonadjacent vertices x and y in G. Show that G contains a Hamiltonian path.

(3) (5%) Let G be a graph of order n such that $|E(G)| \geq \binom{n-1}{2}+1$.

Show that G contains a Hamiltonian path.

2. (10%) $t_1, t_2, \cdots\cdots, t_n \in \mathrm{N}$ (n≥2), $t_1+t_2+\cdots\cdots+t_n=2(n-1)$.

Show that there exists a tree with $t_1, t_2, \cdots\cdots, t_n$ as its degree sequence.

3. (10%) Let T be a tree of order m, and G be a graph with minimum degree $\delta(G) \geq m-1$. Show that G contains a subgraph isomorphic to T.

4. (5%) G is a k-regular bipartite graph (k≥1). Use Hall Theorem to show that G contains a perfect matching.

5. (10%) Let G be a graph such that every two odd cycles in G have at least one common vertex. Show that the chromatic number $\chi(G) \leq 5$.

6. (1) (5%) Let G be a connected plane graph with n vertices, e edges and f faces. Show that n - e +f=2.

(2) (5%) Let G be a planar graph with at least three vertices. Show that $|E(G)| \leq 3|V(G)| - 6$.

(3) (5%) Show that every planar graph contains a vertex with degree $\leq$ 5.

7. (10%) Let G be a graph. A covering of G is a set C of vertices of G such that every edge of G is incident with at least one vertex in C.

Let $\alpha(G)$ = the max. number of vertices in an independent set of G, $\beta(G)$ = the min. number of vertices in a covering of G. Show that $\alpha(G)+\beta(G)=|V(G)|$.

8. (15%) Let G be a graph of order n.

Shat that (1) $\chi(G)+\chi(G^C) \leq n+1$, (2) $\chi(G)\cdot\chi(G^C) \leq \left(\frac{n+1}{2}\right)^2$.