國立中央大學數學系
博士班資格考試
〈機率論〉試題
2002 年 2 月

考試時間三小時。共計十題，每題十分。

(1) Let F_0 be a field and G be the minimal M.C. (Monotone Class) containing F_0.
Define two classes of sets as follows:
$C_1 = \{E \in G : E \cap F \in G \text{ for all } F \in F_0\}$,
$C_2 = \{E \in G : E \cap F \in G \text{ for all } F \in G\}$.
Show that both C_1 and C_2 are M.C.'s and equal to G.

(2) Let $\{X_n\}$ be a sequence of identically distributed random variables with finite mean. Show that $\lim_n \frac{1}{n} E\{\max_{1\le j\le n} |X_j|\} = 0$.

(3) Show that $\{X_n\}$ converges in probability to X if and only if for every subsequence $\{X_{n_k}\}$ contains a further subsequence that converges almost surely to X.

(4) Let $\{X_n\}$ be a sequence of random variables such that $\sup_n E(|X_n|^p) < \infty$ for some $p > 1$. Show that $\{X_n\}$ is uniformly integrable.

(5) Let $\{X_n\}$ be a sequence of independent, identically distributed random variables taking values ± 1 with probability $\frac{1}{2}$ each. Find the range of θ such that $\sum_n \frac{X_n}{n^\theta}$ converges almost surely, and give the explanation.

(6) Let $\{X_n\}$ be a sequence of independent, identically distributed random variables with $EX_n = 0$ and $EX_n^2 = \sigma^2$. Show that $\dfrac{\sum_{i=1}^n X_i}{\left(\sum_{i=1}^n X_i^2\right)^{\frac{1}{2}}}$ converges in distribution to the standard normal distribution.