(7) Let $\{X_{nj}, 1 \le j \le k_n\}$ be a double array of rowwise independent random variables, where $k_n \to \infty$ as $n \to \infty$. Suppose that for each n and j there is a finite constant M_{nj} such that $|X_{nj}| \le M_{nj}$ and that $\max_{1 \le j \le k_n} M_{nj} \to 0$. Show that $\frac{S_n - ES_n}{\sigma_n}$ converges in distribution to the standard normal distribution, where $S_n = X_{n1} + X_{n2} + \ldots + X_{nk_n}$ and $\sigma_n^2 = \{\mathrm{Van}(S_n)\}$.

(8) Let $\{X_n\}$ be a sequence of independent, identically distributed random variables and $N(\lambda)$ be an independent Poisson random variables with mean λ. Show that $W = X_1 + X_2 + \ldots + X_{N(\lambda)}$ has an infinitely divisible distribution.

(9) Let X be a random variable on Ω the probability space, let $A_1, A_2, \ldots, A_n$ be a partition of Ω into disjoint sets each of which has positive probability, and let $F = \sigma(A_1, A_2, \ldots, A_n)$ be the σ field of generated by these sets. Find $E(X|F)$ the conditional expectation of X given F, and give the explanation.

(10) Let $\xi_i^n, i, n \ge 0$ be independent, identically distributed nonnegative integer valued random variables. Define a sequence Z_n, $n \ge 0$ by $Z_0 = 1$ and $Z_{n+1} = \xi_1^{n+1} + \xi_2^{n+1} + \ldots + \xi_{Z_n}^{n+1}$ if $Z_n > 0$, and $Z_{n+1} = 0$ if $Z_n = 0$.

Let $F_n = \sigma(\xi_i^m : i \ge 1,\ 1 \le m \le n)$ be the σ field of generated by these random variables and $\mu = E\xi_i^m$. Show that $\frac{Z_n}{\mu^n}$ is a martingale with respect to F_n.