

Qualifying Examination 2012
Statistical Inference

1. State (a) Basu's Theorem
(b) Rao-Blackwell Theorem (10%)
2. Let X_1, X_2, \dots, X_n be i.i.d. $n(\theta, 1)$.
(a) Find the UMVUE of θ , θ^2 and θ^3 .
(b) Can they attain the Cramer—Rao lower bound? Why? (20%)
3. Let X_1, X_2, \dots, X_n be i.i.d. $n(\theta, 1)$. $g(\theta) = P(X_1 \leq \xi)$, ξ is a constant. Find the UMVUE of $g(\theta)$. (20%)
4. Let X_1, X_2, \dots, X_n be $U(0, \theta)$, $\theta > 0$,
(a) Find the UMP level α test for testing $H_0: \theta \leq \theta_0$ v. s. $H_1: \theta > \theta_0$.
(b) Find the power function $\beta(\theta)$, show that it is increasing in θ . (20%)
5. Let X_1, X_2, \dots, X_n be i.i.d. $n(\theta, 1)$. Find the UMP unbiased α -level test for testing $H_0: \theta \in [\theta_1, \theta_2]$ v.s. $H_1: \theta \notin [\theta_1, \theta_2]$ (10%)
6. Let X_1, X_2, \dots, X_n be i.i.d. beta $(\theta, 1)$, assume θ has a gamma (γ, λ) prior pdf. Find a $1-\alpha$ Bayes credit set for θ . (10%)
7. Find a pivotal quantity based on a random sample of size n from a $n(\theta, \theta)$ population, where $\theta > 0$. Use the pivotal quantity to set up a $1-\alpha$ confidence interval for θ . (10%)