

科目 微積分 類組別 A2,A3,A4,A5,A6,A10 共 1 頁第 1 頁 *請在試卷答案卷(卡)內作答

甲、填充題：共 4 大題，每大題 10 分，共 40 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Let

$$f(x) = \begin{cases} \frac{9x-3\sin 3x}{5x^3}, & x \neq 0 \\ a, & x = 0. \end{cases}$$

If the function f is continuous on the entire real line. Then $a =$ _____.

2. Let

$$f(x, y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

Then $f_{xy}(0, 0) =$ _____ and $f_{yx}(0, 0) =$ _____.

3. Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$. Then the interval of convergence of $f(x)$ is _____, and the interval of convergence of $f'(x)$ is _____.

4. Let R be the region inside the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ where $a, b, c > 0$, and above the plane $z = b - y$, then the volume of the region R is _____.

乙、計算、證明題：共 6 大題，每大題 10 分，共 60 分。須詳細寫出演算過程、否則不予計分。

5. Test the series for convergence or divergence using any appropriate test. Identify the test used and explain your reasoning.

$$(a) \sum_{n=1}^{\infty} n 2^{-n} \sin \frac{1}{n}. \quad (b) \sum_{n=1}^{\infty} \frac{2^n}{2^{n+1} + 1}.$$

6. Find the absolute maximum and minimum values of

$$f(x, y, z) = 2x - 2y + z$$

subject to the constraint $x^2 + y^2 + z^2 = 1$.

7. Evaluate the following integrals.

$$(a) \int \sin(\ln x) dx. \quad (b) \int_0^1 \frac{x^3 - 1}{\ln x} dx. \quad \text{Hint: } \int_0^3 a^x dx = \frac{a^3 - 1}{\ln a} \text{ for } a > 0, a \neq 1.$$

8. Find the following limits.

$$(a) \lim_{x \rightarrow 0^+} x \int_x^1 \frac{\cos t}{t^2} dt. \quad (b) \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\tan x}.$$

9. Show that

$$0 < \int_{-\frac{1}{\sqrt{2}}}^1 \frac{\sin(\pi x^2)}{x} dx < \frac{1}{2} \ln 2.$$

10 Consider the function $f(x, y) = \alpha x^2 + \beta y^2$. Find values for α and β so that (a) $(0, 0)$ is a local minimum (b) $(0, 0)$ is a local maximum, and (c) $(0, 0)$ is a saddle.