

# On The Study of Travelling Wave

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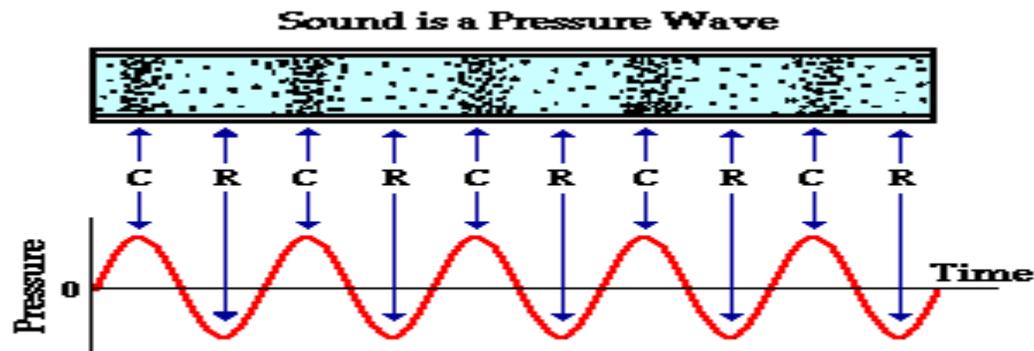
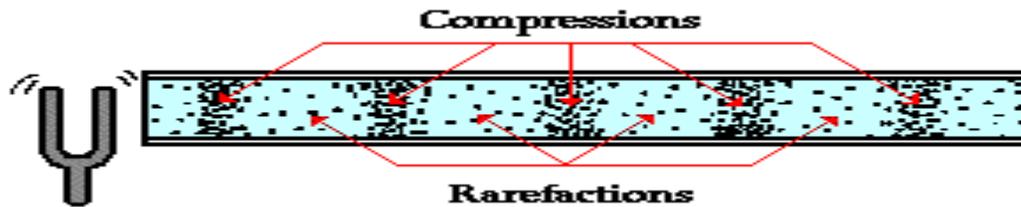
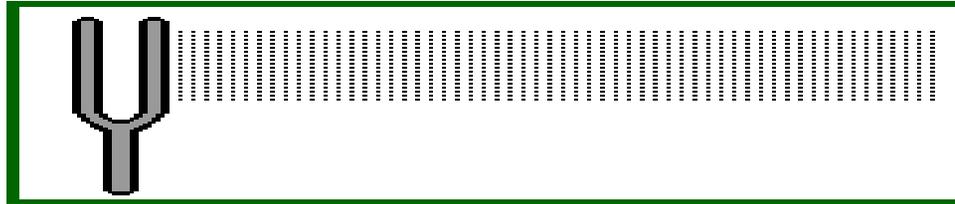
# ***A Travelling Wave***



# Examples of traveling waves

- sound waves
- water waves on the surface of the ocean
- seismic waves

# Sound is a Pressure Wave

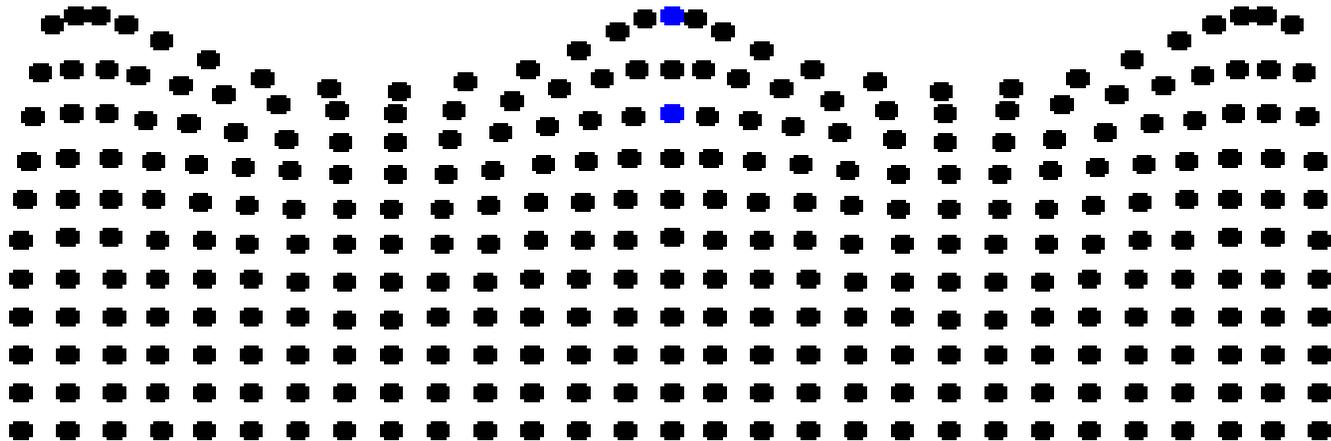


NOTE: "C" stands for compression and "R" stands for rarefaction

# Water Wave



# Water Waves



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# Seismic wave

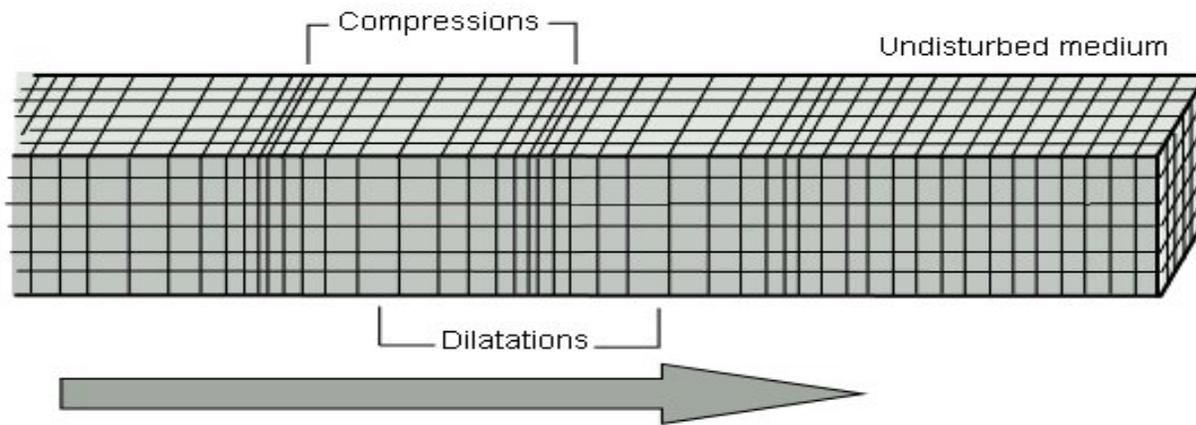
- Energy wave generated by an [earthquake](#) or an artificial explosion.
- Two types of seismic waves: **body waves** that travel through the Earth's interior; and **surface waves** that travel through the surface layers of the crust and can be felt as the shaking of the ground, as in an earthquake.

# Types of earthquake waves

- P, or primary, waves, which are compressional and travel fastest;
- S, or secondary, waves, which are transverse, i.e., they cause the earth to vibrate perpendicularly to the direction of their motion.
- Surface waves consist of several major types and are called L, or long, waves.
- Since the velocities of the P and S waves are affected by changes in the density and rigidity of the material through which they pass, the boundaries between the regions of the earth known as the crust, mantle, and core have been discerned by seismologists, scientists who deal with the analysis and interpretation of earthquake waves (see [earth](#)). Seismographs (see [seismology](#)) are used to record P, S, and L waves. The disappearance of S waves below depths of 1,800 mi (2,900 km) indicates that at least the outer part of the earth's core is liquid

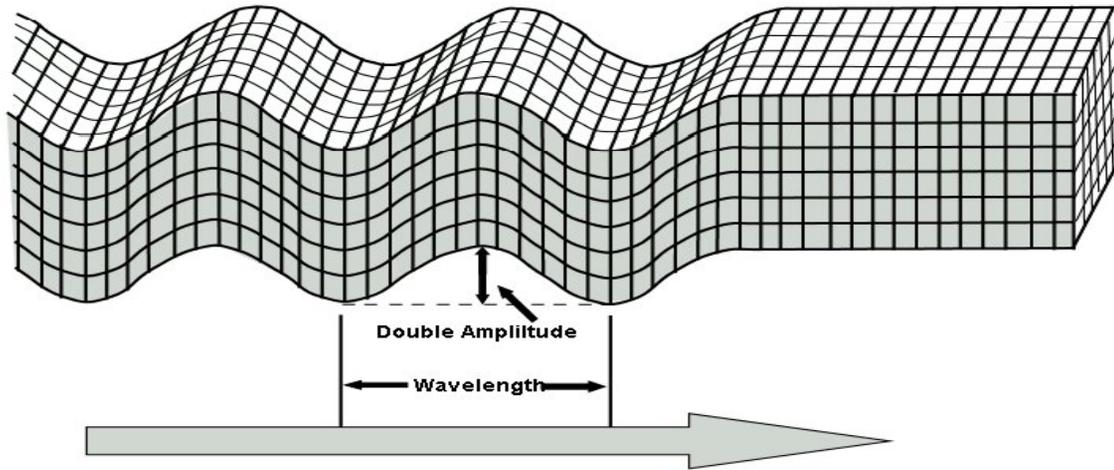
# P-Wave

**P Wave**



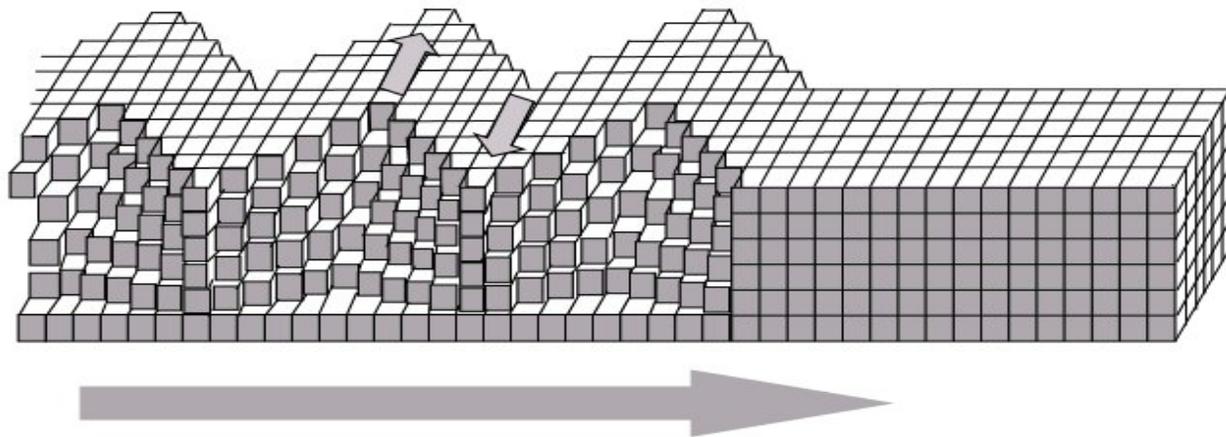
# S-Wave

S Wave



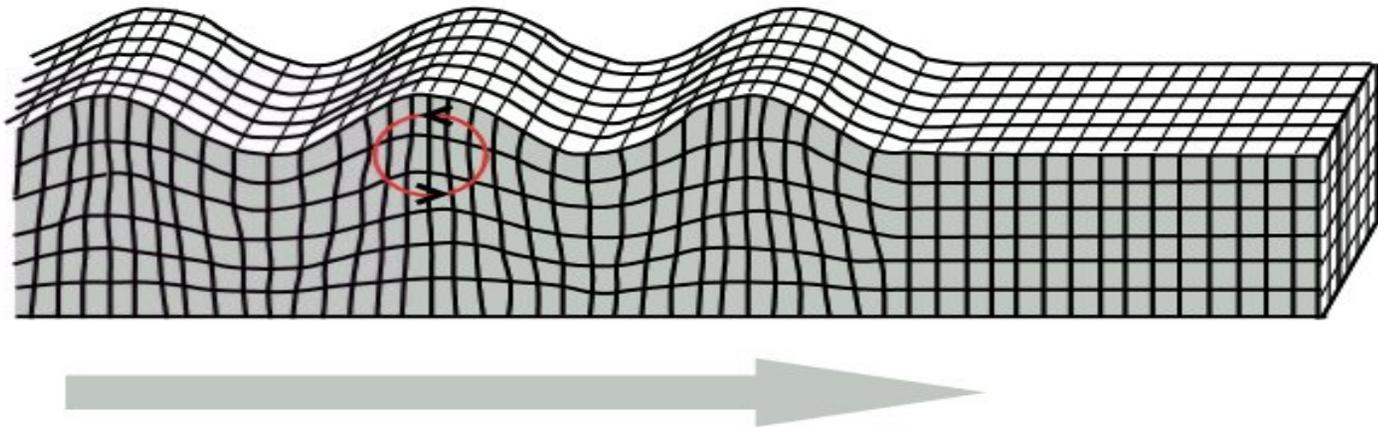
# Love-Wave

Love Wave

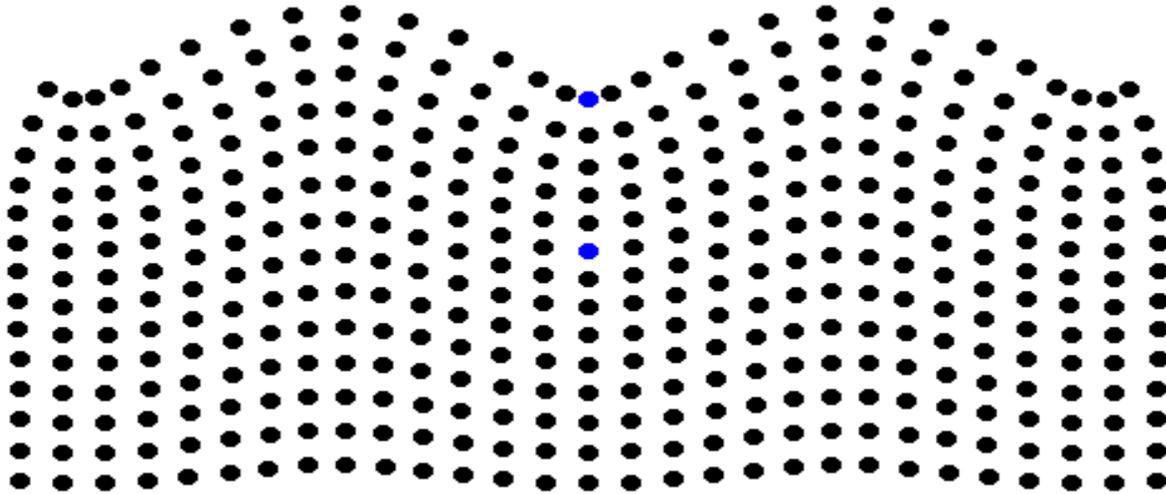


# Rayleigh Wave

Rayleigh Wave



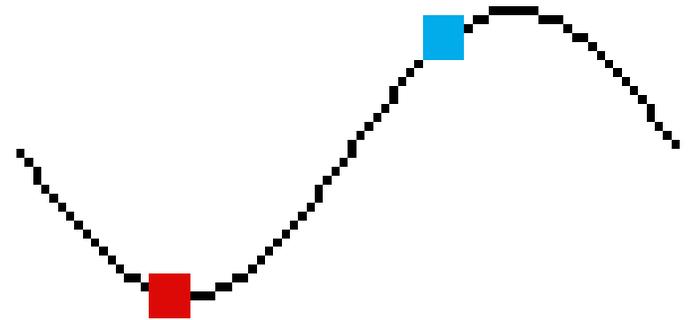
# Rayleigh surface waves



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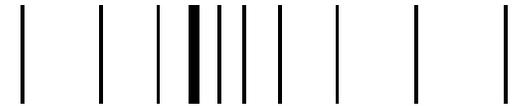
# Demonstration of Transverse and Longitudinal Waves

- This is a **transverse wave** wave, such as a wave on a string. The wave is moving to the right, but the individual points on the string, such as those indicated by red and blue dots, are moving up and down (perpendicular to the direction of motion of the string). You can convince yourself that the red and blue dots are really moving up and down (not backward or forward) by positioning the cursor just below one of them.



# Longitudinal wave

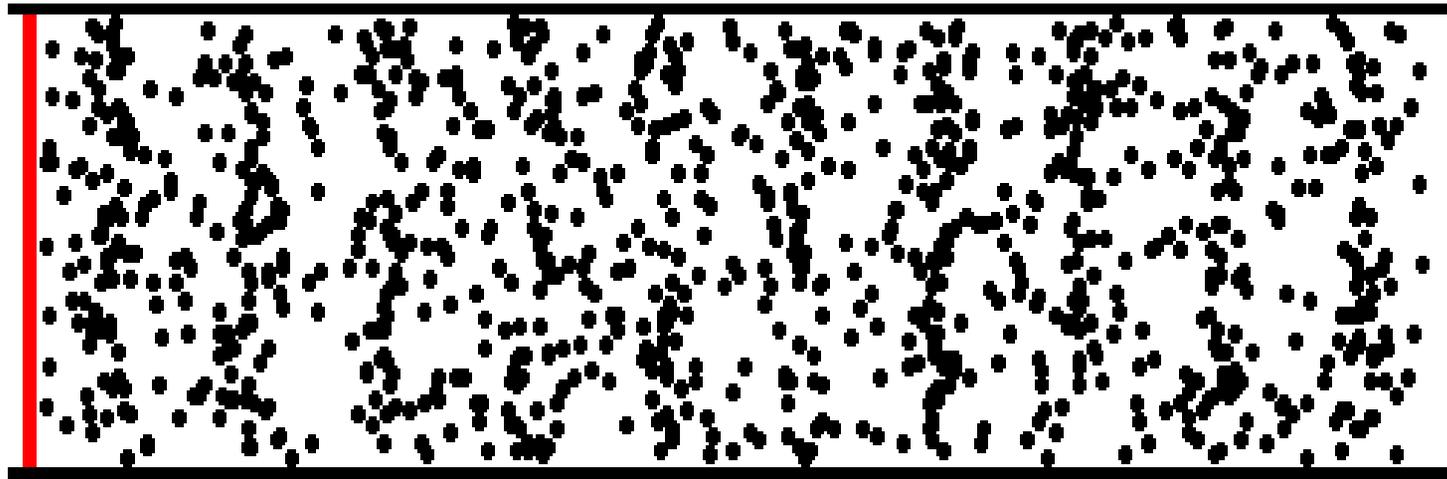
- This is a **longitudinal wave**, such as a compression wave in a fluid. This wave is also moving to the right, as you can see by noting that the region of compression (where the particles are close together) gradually moves to the right. But each individual particle moves back and forth parallel to the motion of the wave itself.



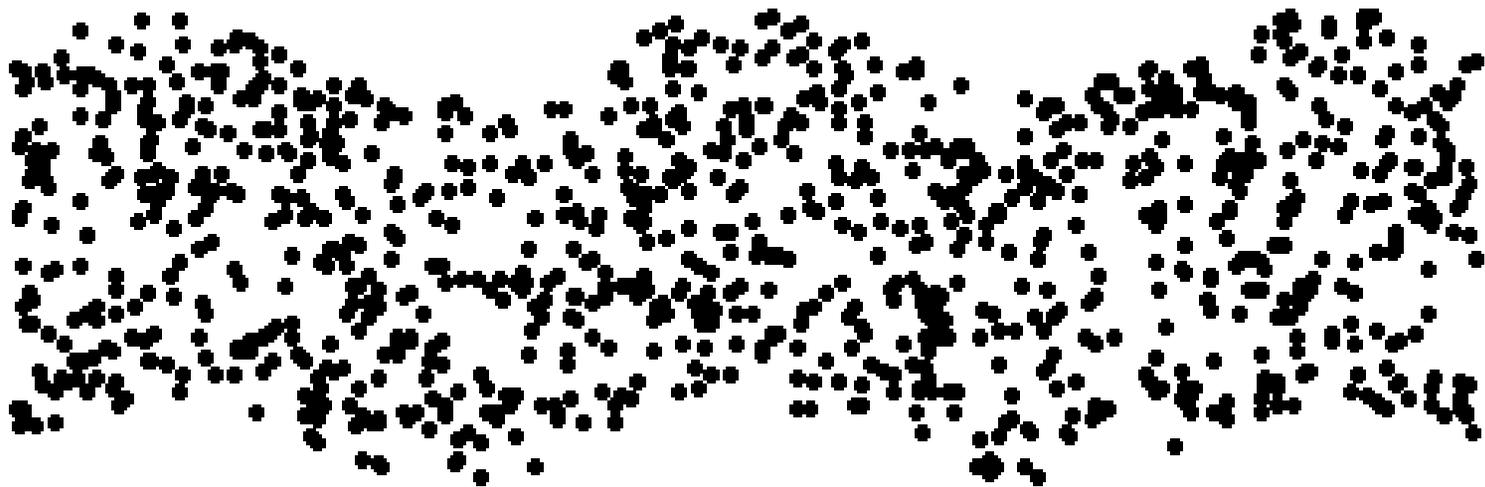
# Longitudinal wave

- In a longitudinal wave the particle displacement is parallel to the direction of wave propagation. The animation below shows a one-dimensional longitudinal plane wave propagating down a tube. The particles do not move down the tube with the wave; they simply oscillate back and forth about their individual equilibrium positions. Pick a single particle and watch its motion. The wave is seen as the motion of the compressed region (ie, it is a pressure wave), which moves from left to right.

# Longitudinal wave



# Transverse Waves



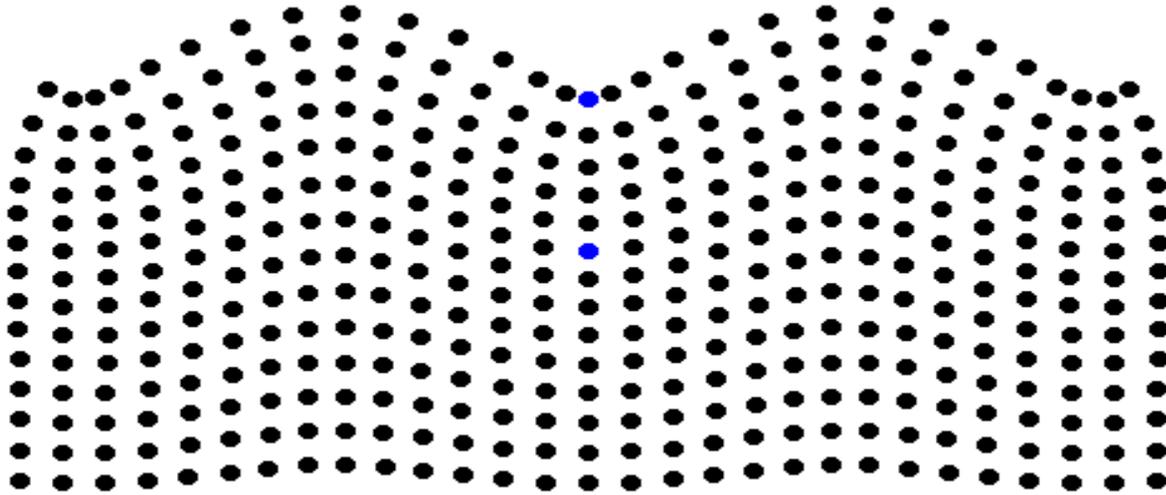
# Transverse Waves

- In a transverse wave the particle displacement is perpendicular to the direction of wave propagation. The animation below shows a one-dimensional transverse plane wave propagating from left to right. The particles do not move along with the wave; they simply oscillate up and down about their individual equilibrium positions as the wave passes by. Pick a single particle and watch its motion

# Rayleigh surface waves

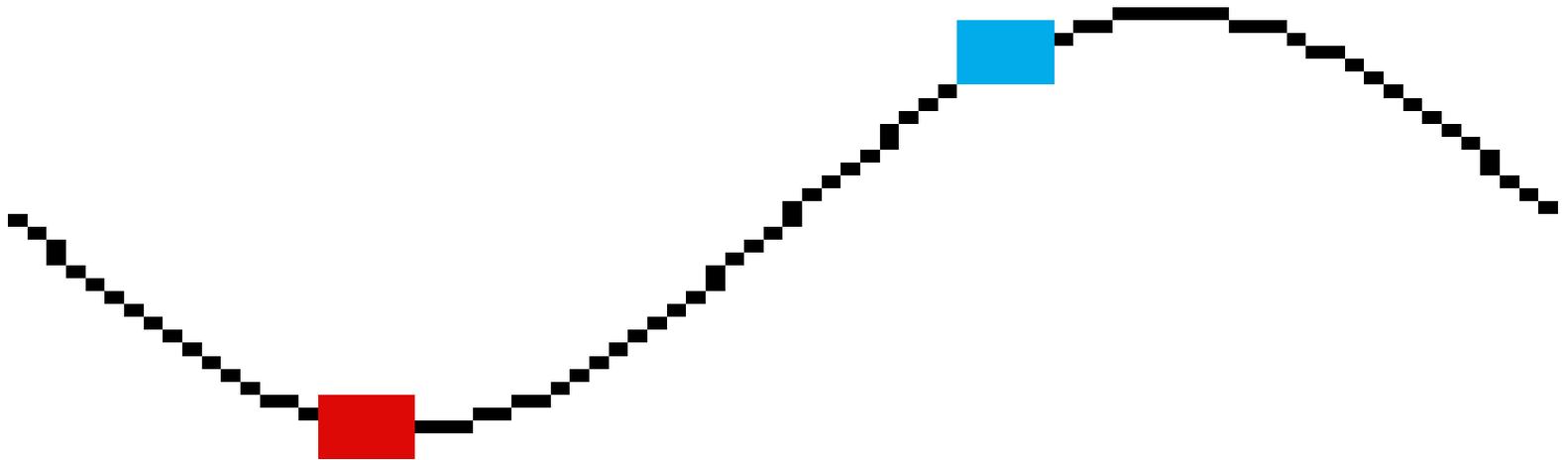
- Another example of waves with both longitudinal and transverse motion may be found in solids as **Rayleigh surface waves**. The particles in a solid, through which a Rayleigh surface wave passes, move in elliptical paths, with the major axis of the ellipse perpendicular to the surface of the solid. As the depth into the solid increases the "width" of the elliptical path decreases. Rayleigh waves are different from water waves in one important way. In a water wave all particles travel in clockwise circles. However, in a Rayleigh surface wave, particles at the surface trace out a *counterclockwise* ellipse, while particles at a depth of more than  $1/5$ th of a wavelength trace out *clockwise* ellipses. The movie below shows a Rayleigh wave travelling from left to right along the surface of a solid. I have identified two particles in **blue** to illustrate the counterclockwise-clockwise motion as a function of depth

# Rayleigh surface waves

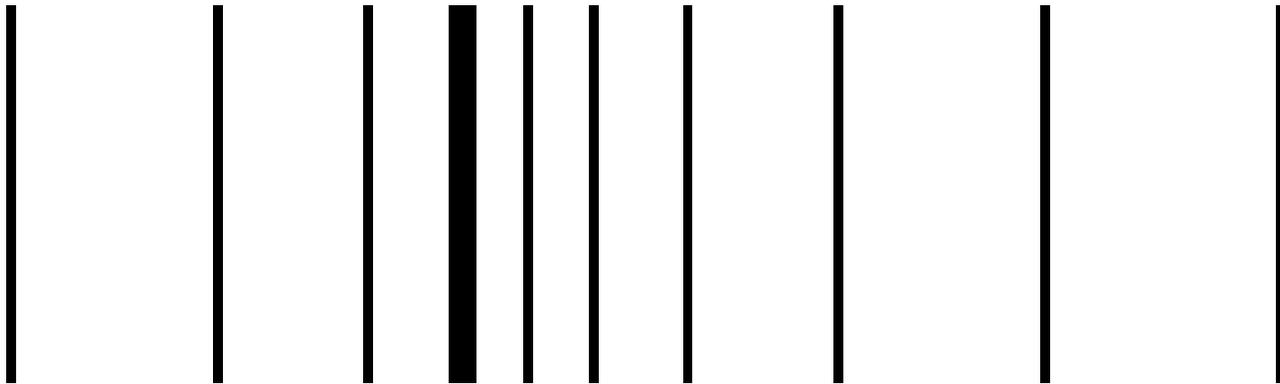


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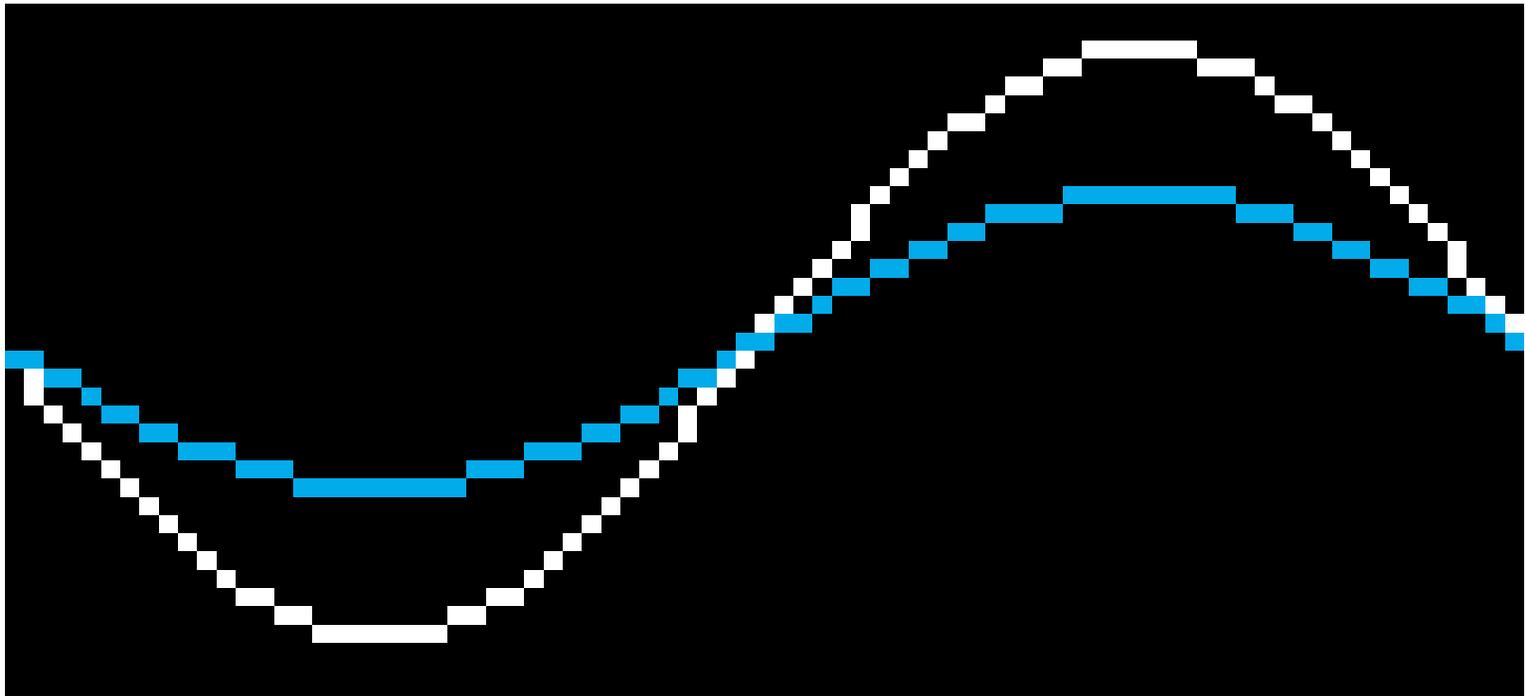
# Transverse travelling wave



# Longitudinal travelling wave



# Formation of transverse standing wave by superposition



# Waves in Ideal String

- If a constant horizontal tension  $T$  is maintained in the string, then

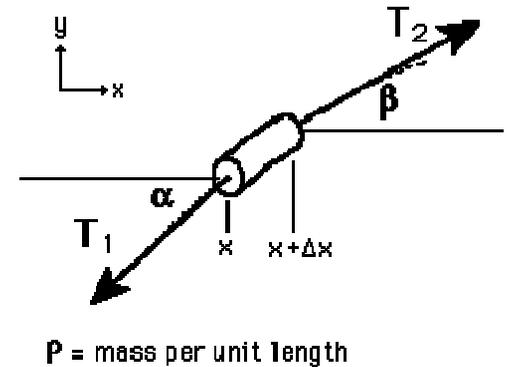
$$T_1 \cos \alpha = T_2 \cos \beta = T$$

- Newton's 2nd law becomes

$$T_2 \sin \beta - T_1 \sin \alpha = \rho \Delta x \frac{\partial^2 y}{\partial t^2}$$

- Combination of these two expressions for small angles gives

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}$$



# Wave Equation

- The wave equation for a plane wave traveling in the x direction is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

- where  $v$  is the phase velocity of the wave and  $y$  represents the variable which is changing as the wave passes. This is the form of the wave equation which applies to a stretched string or a plane electromagnetic wave. In two dimensions, the wave equation takes the form

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$$

which could describe a wave on a stretched membrane

# Traveling Wave Solution for String (I)

- A solution to the wave equation for an ideal string can take the form of a traveling wave

$$y(x,t) = A \sin \frac{2\pi}{\lambda} (x - vt) \quad \text{or} \quad y(x,t) = A \sin \frac{2\pi}{\lambda} (x + vt)$$

- For a string of length  $L$  which is fixed at both ends, the solution can take the form of standing waves:  
$$y(x,t) = A \sin \omega_n t \sin \frac{n\pi x}{L}$$

# Traveling Wave Solution for String (II)

$$y(x,t) = A \sin \frac{2\pi}{\lambda} (x - vt) \quad \text{or} \quad y(x,t) = A \sin \frac{2\pi}{\lambda} (x + vt)$$

is a solution of

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial y}{\partial x} = \frac{2\pi A}{\lambda} \cos \frac{2\pi}{\lambda} (x + vt)$$

$$\frac{\rho}{T} \frac{\partial y}{\partial t} = \frac{\rho}{T} \frac{2\pi v A}{\lambda} \cos \frac{2\pi}{\lambda} (x + vt)$$

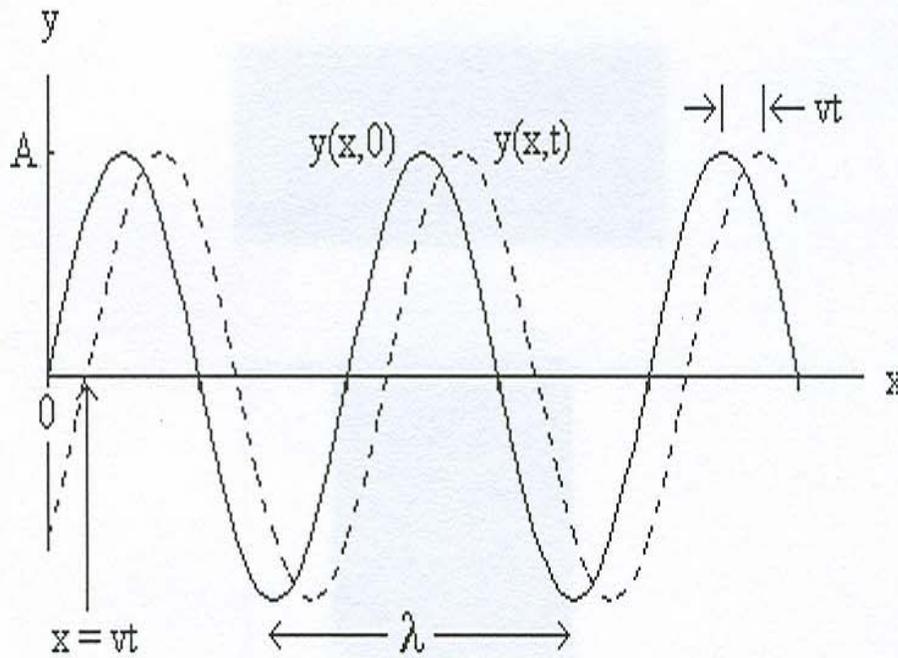
$$\frac{\partial^2 y}{\partial x^2} = \frac{-4\pi^2}{\lambda^2} A \sin \frac{2\pi}{\lambda} (x + vt)$$

$$\frac{\rho}{T} \frac{\partial^2 y}{\partial t^2} = \frac{\rho}{T} \frac{-4\pi^2 v^2 A}{\lambda^2} \sin \frac{2\pi}{\lambda} (x + vt)$$

- $v = \sqrt{\frac{T}{\rho}}$  : Wave velocity for a stretched string

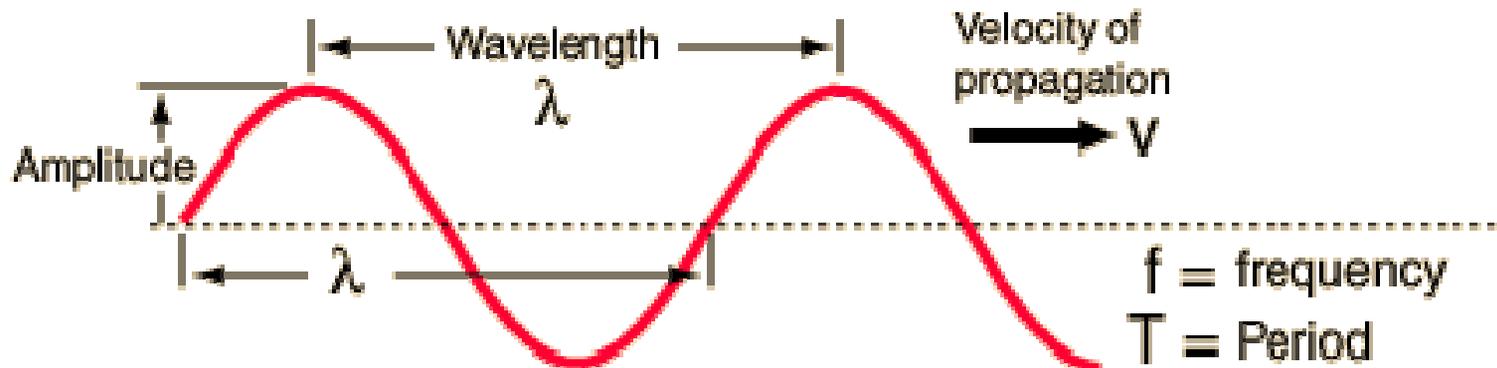
# Wave

The diagram shows a snapshot of a travelling wave at two instants of time 0 and t.



# Traveling Wave Relationship

- A single frequency traveling wave will take the form of a sine wave. A snapshot of the wave in space at an instant of time can be used to show the relationship of the wave properties frequency, wavelength and propagation velocity



# Two sine waves travelling in the same direction: Constructive and Destructive Interference

- Two waves (with the same amplitude, frequency, and wavelength) are travelling in the same direction on a string. Using the principle of superposition, the resulting string displacement may be written as:

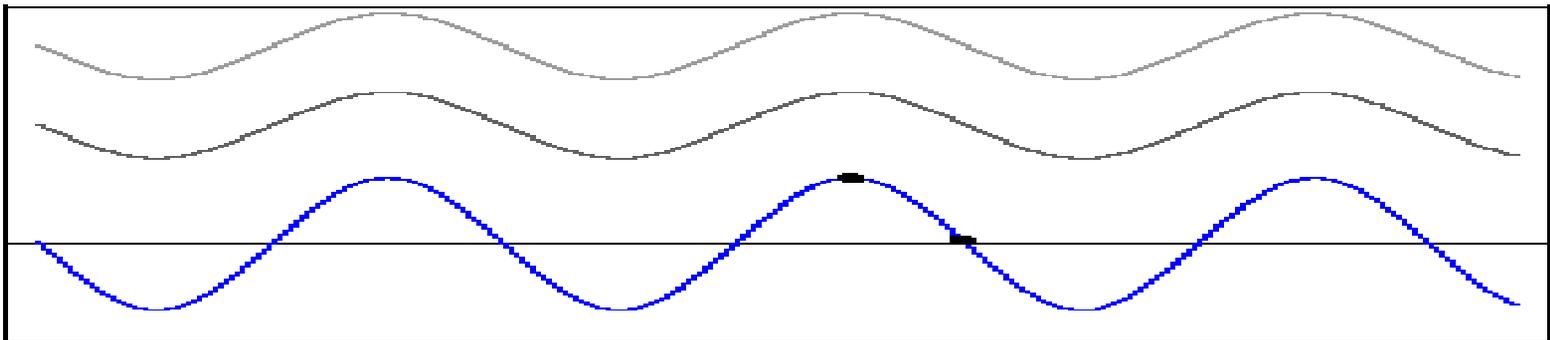
$$\begin{aligned}y(x, t) &= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \\ &= 2y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)\end{aligned}$$

- which is a travelling wave whose amplitude depends on the phase ( $\phi$ ). When the two waves are **in-phase** ( $\phi=0$ ), they interfere **constructively** and the result has twice the amplitude of the individual waves. When the two waves have **opposite-phase** ( $\phi=180$ ), they interfere **destructively** and cancel each other out.

# Two sine waves travelling in the same direction:

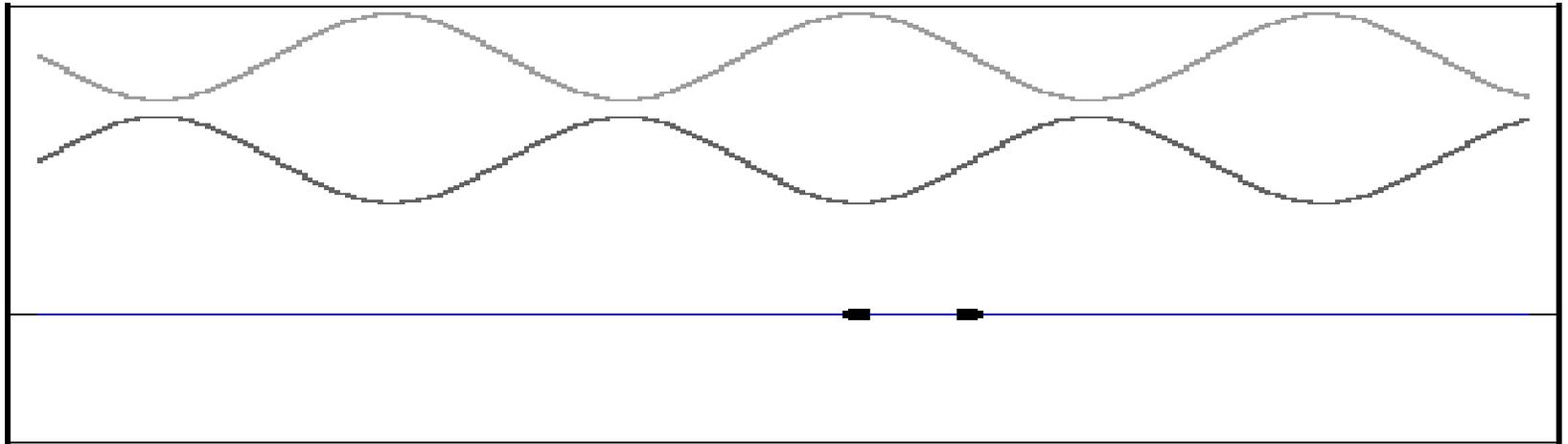
- Constructive and Destructive Interference

$$\begin{aligned}y(x, t) &= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \\ &= 2y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)\end{aligned}$$



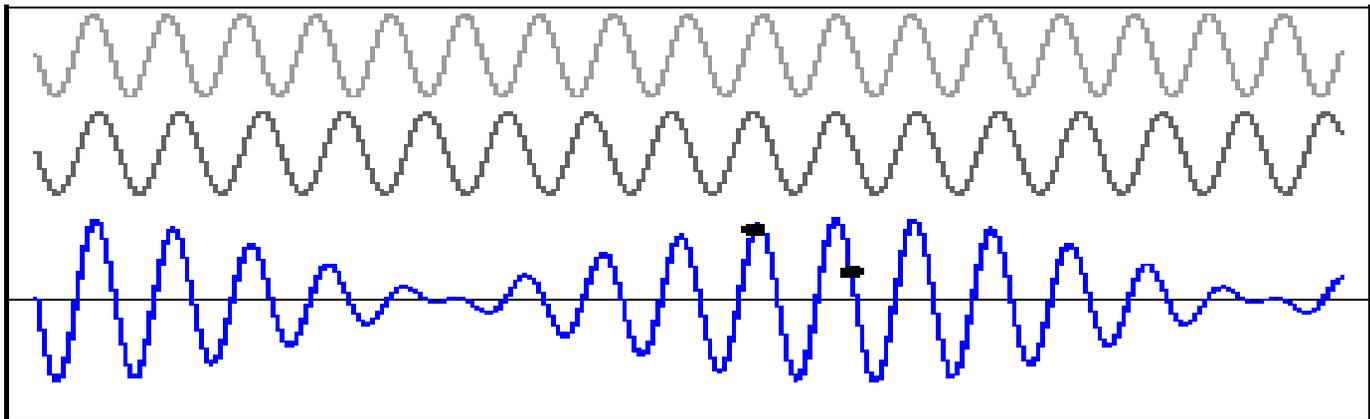
Two sine waves travelling in opposite directions create a standing wave

$$\begin{aligned}y(x, t) &= y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) \\ &= 2y_m \sin kx \cos \omega t\end{aligned}$$



# Two sine waves with different frequencies: Beats

$$\begin{aligned}y(x, t) &= y_m \sin(k_1 x - \omega_1 t) + y_m \sin(k_2 x - \omega_2 t) \\ &= 2y_m \cos\left[\frac{(k_1 - k_2)}{2}x - \frac{(\omega_1 - \omega_2)}{2}t\right] \sin\left[\frac{(k_1 + k_2)}{2}x - \frac{(\omega_1 + \omega_2)}{2}t\right]\end{aligned}$$



Thank You !