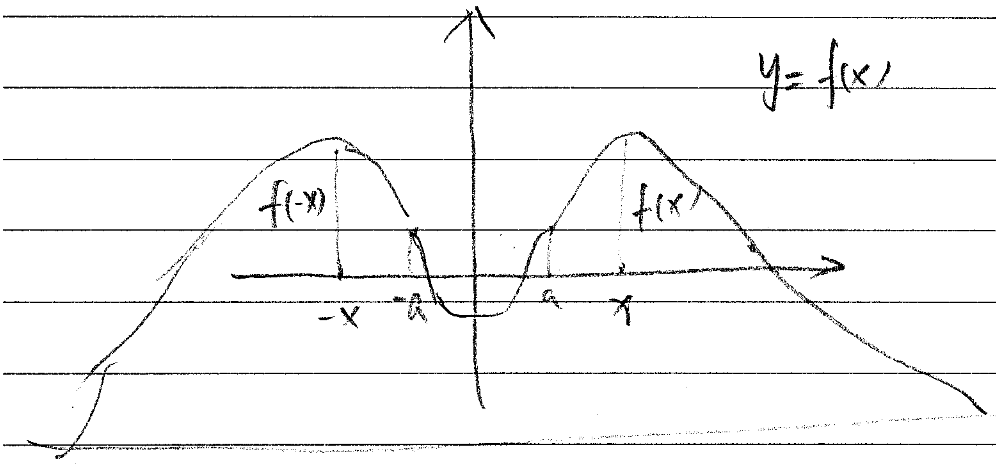
自我練習

Def $f: R \to R$

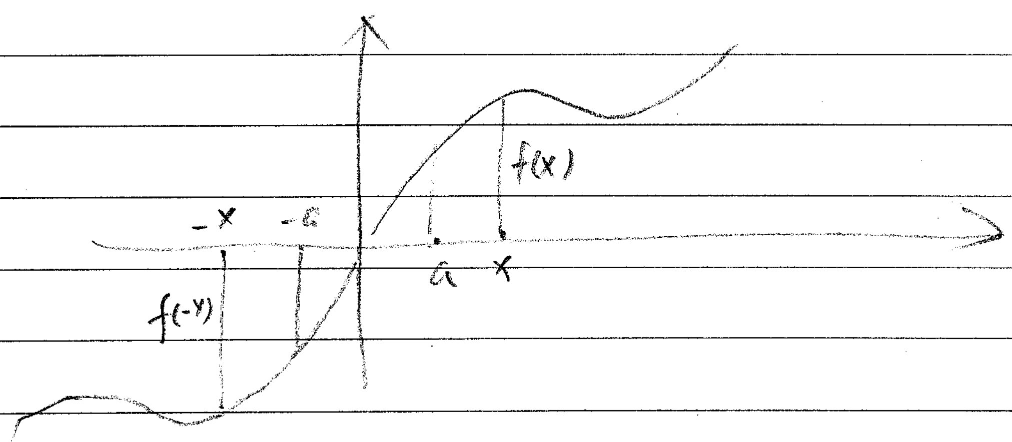
① If $f(-x) = f(x)$ for all $x \in R$, then f is an <u>even function</u>

② If $f(-x) = -f(x)$ for all $x \in R$, then f is an <u>odd function</u>

Note 1. f is an even function $\Rightarrow$ graph of f is symmetric
(f之圖形)
with respect to Y axis



Note 2. f is an odd function $\Rightarrow$ graph of f is symmetric with respect to the origin



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Problem.

$f: R \to R$, $g: R \to R$. Prove each of the following.

1. If f is an even function, g is an odd function then $f \cdot g$ is an odd function
2. If f is an even function, then $g \circ f$ is an even function
3. If f is an odd function, g is an odd function then $g \circ f$ is an odd function
4. If f is an odd function g is an even function then $g \circ f$ is an even function.

Sol. 1. $(f \cdot g)(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-g(x))$ ($\because$ f is even, g is odd)

$= -f(x) \cdot g(x) = -(f \cdot g)(x)$

$\therefore$ $f \cdot g$ is odd

2. $(g \circ f)(-x) = g(f(-x)) = g(f(x))$ ($\because$ f is even)

$= (g \circ f)(x)$

$\therefore$ $g \circ f$ is even

3. $(g \circ f)(-x) = g(f(-x)) = g(-f(x))$ ($\because$ f is odd)

$= -g(f(x))$ ($\because$ g is odd)

$= -(g \circ f)(x)$

$\therefore$ $g \circ f$ is odd.

4. $(g \circ f)(-x) = g(f(-x)) = g(-f(x))$ ($\because$ f is odd)

$= g(f(x))$ ($\because$ g is even)

$= (g \circ f)(x)$

$\therefore$ $g \circ f$ is even