

Calculus I Midterm 1 Sample, the rest 40%

National Central University, Summer 2011

Problem 1. Suppose that $f(x) = -(2 + \frac{\pi}{4}) \sin x + x \cos x$ is defined on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

(1) Show that

$$|f(x) - f(y)| \leq (1 + \frac{\pi}{2}) \frac{\sqrt{2}}{2} |x - y| \quad \forall x, y \in [-\frac{\pi}{2}, \frac{\pi}{2}].$$

(2) Sketch the graph of f defined in (2) with the information of

- (a) intercepts;
- (b) interval of increase and decrease;
- (c) extreme values and critical points; and
- (d) concavity and inflection points.

Problem 2. Exercise problem 70 in Section 4.7.

Problem 3. Show that $x^3 + x + 1 = 0$ has exactly one solution, and use Newton's method to compute the approximated solution x_2 with the initial guess $x_0 = 0$.

Problem 4. Find an anti-derivative of $g(x) = x \sin x$.

Hint: Check the derivative of f defined in Problem 1 (2).