Calculus MA1002-A Quiz 10

National Central University, June 13 2019

學號：____________________ 姓名：____________________

Problem 1. (5%) Rewrite the iterated integral $\int_0^4 \Big[\int_0^{\frac{4-x}{2}} \Big(\int_0^{\frac{12-3x-6y}{4}} dz\Big)dy\Big]dx$ in the order $dydxdz$.

Solution. Let Q be the tetrahedron（四面體）with vertices $(0,0,0)$, $(4,0,0)$, $(0,2,0)$ and $(0,0,3)$ (which is the region bounded by $x=0, y=0, z=0$ and $z=\dfrac{12-3x-6y}{4}$). Then

$$\iiint_Q dV = \int_0^4 \Big[\int_0^{\frac{4-x}{2}} \Big(\int_0^{\frac{12-3x-6y}{4}} dz\Big)dy\Big]dx\,.$$

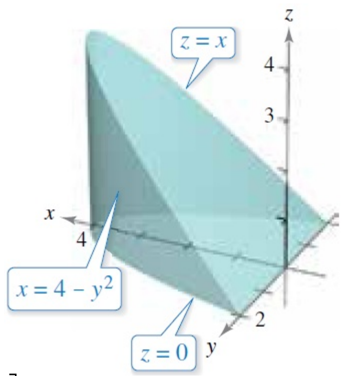
Let R be the projection of Q along the y-axis onto xz-plane. Then

$$R = \Big\{(x,z)\,\Big|\, 0\leqslant z\leqslant 3, 0\leqslant x\leqslant \frac{12-4z}{3}\Big\}\,;$$

thus the Fubini Theorem implies that

$$\iiint_Q dV = \int_0^3 \Big[\int_0^{\frac{12-4z}{3}} \Big(\int_0^{\frac{12-3x-4z}{6}} dy\Big)dx\Big]dz\,.$$ □

Problem 2. (5%) Find the volume of the solid region shown in the figure below using triple integrals in the order $dxdydz$.



Solution. Let Q denote the shaded solid region in the figure, and R be the projection of Q along the x-axis onto the yz-plane. Then R is the region (on the yz-plane) bounded by the y-axis and the graph of $z = 4-y^2$. Therefore,

$$R = \big\{(y,z)\,\big|\, 0\leqslant z\leqslant 4, -\sqrt{4-z}\leqslant y\leqslant \sqrt{4-z}\big\}\,,$$

and the Fubini Theorem implies that the volume of Q is given by

$$\iiint_Q dV = \int_0^4 \Big[\int_{-\sqrt{4-z}}^{\sqrt{4-z}} \Big(\int_z^{4-y^2} dx\Big)\,dy\Big]dz = \int_0^4 \Big[\int_{-\sqrt{4-z}}^{\sqrt{4-z}} (4-y^2-z)\,dy\Big]dz$$

$$= \int_0^4 \Big[2(4-z)\sqrt{4-z} - \frac{2}{3}(4-z)^{\frac{3}{2}}\Big]\,dz = \int_0^4 \frac{4}{3}(4-z)^{\frac{3}{2}}\,dz = -\frac{8}{15}(4-z)^{\frac{5}{2}}\Big|_{z=0}^{z=4}$$

$$= \frac{256}{15}\,.$$ □