微積分 MA1003-I 班加分項目一

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Problem 1. Let $a > 0$ and $n \in \mathbb{N}$. Show that $\lim_{x\to a} x^{\frac{1}{n}} = a^{\frac{1}{n}}$ using the $\varepsilon - \delta$ definition.

Hint: For a given $\varepsilon > 0$, choose $\delta = \min\left\{\frac{a}{2}, \frac{na^{\frac{n-1}{n}}\varepsilon}{2}\right\}$. Clearly $\delta > 0$. Show that

$$\text{if}\ \ 0 < |x-a| < \delta\ \ \text{then}\ \ |x^{\frac{1}{n}} - a^{\frac{1}{n}}| < \varepsilon.$$

You may also want to use the identity $(a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1}) = a^n - b^n$.