Exercise Problem Sets 3

Apr. 16 2020

(due Apr. 22 2020)

Problem 1. In this problem we find the five-point formula for $f'(x_0)$. In class we have shown that given the value of f at $(n+1)$-points $x_0, x_1, \cdots, x_n$, under suitable condition for each $j \in \{0, 1, 2, \cdots, n\}$ there exists ξ between $\min\{x_0, x_1, \cdots, x_n\}$ and $\max\{x_0, x_1, \cdots, x_n\}$ such that

$$f'(x_j) = \sum_{k=0}^{n} f(x_k) L'_{n,k}(x_j) + \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0, i\neq j}^{n} (x_j - x_i) \,.$$

Complete the following.

1. Show that $L'_{n,k}(x_j) = \begin{cases} \dfrac{\prod_{i=0, i\neq k,j}^{n} (x_j - x_i)}{\prod_{i=0, i\neq k}^{n} (x_k - x_i)} & \text{if } j \neq k \,, \\ \displaystyle\sum_{i=0}^{n} \frac{1}{x_k - x_i} & \text{if } j = k \,. \end{cases}$

2. Suppose that $h > 0$ and $x_k = x_0 + kh$, $k = 1, 2, 3, 4$. Show that

$$f'(x_0) = \frac{-25 f(x_0) + 48 f(x_0 + h) - 36 f(x_0 + 2h) + 16 f(x_0 + 3h) - 3 f(x_0 + 4h)}{12h} + \frac{h^4}{5} f^{(5)}(\xi)$$

for some ξ between x_0 and $x_0 + 4h$, and

$$f'(x_2) = \frac{f(x_2 - 2h) - 8 f(x_2 - h) + 8 f(x_2 + h) - f(x_2 + 2h)}{12h} + \frac{h^4}{30} f^{(5)}(\eta)$$

for some η between $x_2 - 2h$ and $x_2 + 2h$.