# 最佳化方法與應用 MA5037

## 期中考加分題

Due Dec. 06. 2023

**Problem 1.** Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function given by

$$f(x, y) = xe^{-x^2-y^2}$$

whose gradient and Hessian are given respectively by

$$(\nabla f)(x, y) = e^{-x^2-y^2}\big[1 - 2x^2; -2xy\big]$$

and

$$(\nabla^2 f)(x, y) = e^{-x^2-y^2}\begin{bmatrix} 4x^3 - 6x & 4x^2y - 2y \\ 4x^2y - 2y & 4xy^2 - 2x \end{bmatrix}.$$

1. At the $k$-th iterate $x_k$, let

   $$B_k = (\nabla^2 f)(x_k) + \delta_k \mathrm{I}_2\,,$$

   where $\delta_k \geqslant 0$ is chosen so that the minimal eigenvalue of $B_k$ is not less then $10^{-5}$. Write a matlab® function named B_k to generate such $B_k$ for a given $x_k$:

   $$B_k = \mathrm{B\_k}(x_k).$$

2. At the $k$-th iterate $x_k$, for a given trust region radius $\Delta_k$ consider solving the problem

   $$\min_{x\in\mathbb{R}^n} m(p) = f(x_k) + \nabla f(x_k)^{\mathrm{T}} p + \frac{1}{2} p^{\mathrm{T}} B_k p \quad \text{subject to} \quad \|p\| \leqslant \Delta_k\,.$$

   Write a matlab® function named Dogleg which implements the Dogleg method of finding an approximated value of the exact minimizer $p_k^*$:

   $$p_k = \mathrm{Dogleg}(x_k, B_k, \Delta_k).$$

3. Write a matlab® function named algorithm41 to implement Algorithm 4.1 (together with Dogleg of obtaining $p_k$) in the textbook/slides:

   $$x_{\text{final}} = \mathrm{algorithm41}(x_0, \Delta_0, \widehat{\Delta}, \eta, \varepsilon, N),$$

   where

   (a) $x_0$ is the initial guess of the minimizer of $f$,

   (b) $\Delta_0$ is the initial guess of the trust region radius,

   (c) $\widehat{\Delta}$ is the upper bound for trust region radius,

   (d) $\eta$ is the parameter which is used to judge if we will step forward ($x_{k+1} = x_k + p_k$),

   (e) $\varepsilon$ and $N$ are the parameters for the stopping criteria: go on to the next step if $k \leqslant N$ and $\|\nabla f(x_k)\|_\infty > \varepsilon\big[1 + f(x_k)\big]$.

   (f) $x_{\text{final}}$ is the last iterate of the sequence.

4. Try various initial guess $x_0$ and $\Delta_0$ (with $\widehat{\Delta} = 10^{-2}$, $\eta = 0.1$, $\varepsilon = 10^{-5}$ and $N = 10^4$) to observe the performance of the trust region method.