管院微積分聯合教學　　會考四　　Mar 21, 2017

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。 (不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

F

1. $\int f'(x)g(h(x))\,dx = f(x)g(h(x)) - \int f(x)g'(h(x))\,dx.$

$$\int f'(x)g(h(x))\,dx = f(x)g(h(x)) - \int f(x)(g\circ h)'(x)\,dx$$

$$= f(x)g(h(x)) - \int f(x)g'(h(x))h'(x)\,dx$$

T

2. The ~~indefinite~~ improper integral $\int_{-\infty}^{-1} x^{-2}\,dx$ converges.

送分

$$\int_{-\infty}^{-1} x^{-2}\,dx = \lim_{a\to-\infty}\int_a^{-1} x^{-2}\,dx = \lim_{a\to-\infty}\left[\frac{-1}{x}\right]_a^{-1}$$

$$= \lim_{a\to-\infty}\left(1-\frac{-1}{a}\right)$$

$$= 1 \qquad \text{since } \lim_{a\to-\infty}\frac{-1}{a}=0$$

So $\int_{-\infty}^{-1} x^{-2}dx$ converges

F **3.** The level surface of $f(x, y, z) = 2x^2 + 3y^2 - z$ that contains the point $(2, -1, 3)$ is $2x^2 + 3y^2 - z = 2$.

$f(2,-1,3) = 2(2)^2 + 3(-1)^2 - 3 = 8 + 3 - 3 = 8 \neq 2.$

F **4.** If $f_x(a, b) = 0 = f_y(a, b)$ and $f_{xx}(a, b)f_{yy}(a, b) < 0$, then $f(a, b)$ is a local extremum of f.

$$D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy}(a,b))^2$$
$$< -(f_{xy}(a,b))^2$$
$$< 0$$

$\Rightarrow f(a,b)$ is not a local extremum by second derivative test.

T **5.** Let $f(x, y) = x^4 + y^4$, then $f(0, 0)$ is the absolute minimum of f.

$f(0,0) = 0 \le x^4 + y^4 = f(x,y) \quad \forall\, x, y \in \mathbb{R}$

$\Rightarrow f(0,0)$ is the absolute minimum of f.

F **6.** Let $f = e^x$. If n is large, then trapezoidal rule is more efficient than Simpson's rule.

Since $f^{(4)}(x) = f'''(x) = f''(x) = f'(x) = f(x) = e^x$ and $e^x \le e^b$ on $[a,b]$. Therefore by error analysis on page 508, we have the maximum error incurred in using trapezoidal rule and Simpson's rule are $\frac{e^b(b-a)^3}{12n^2}$ and $\frac{e^b(b-a)^5}{180n^4}$ respectively. n^4 grows faster than n^2, so Simpson's rule is more efficient than trapezoidal rule.

F **7.** $\int_{-\infty}^{\infty} \frac{x}{1+x^2} dx = 0.$

$$\int_0^{\infty} \frac{x}{1+x^2} dx = \lim_{b\to\infty} \int_0^b \frac{x}{1+x^2} dx = \lim_{b\to\infty} \frac{1}{2} \ln |1+b^2| = \infty$$

So $\int_0^{\infty} \frac{x}{1+x^2} dx$ is divergent, therefore

$\int_{-\infty}^{\infty} \frac{x}{1+x^2} dx = \int_0^{\infty} \frac{x}{1+x^2} dx + \int_{-\infty}^{0} \frac{x}{1+x^2} dx$ is also divergent.

T **8.** If $f(x, y) = x^2y + e^{x^2+y^2}$, then

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(x^2 + 2y e^{x^2+y^2}\right) = 2x + 4xy e^{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(2xy + 2x e^{x^2+y^2}\right) = 2x + 4xy e^{x^2+y^2}$$

F **9.** If f does NOT have an extrema at (a, b), then (a, b) is a saddle point.

Since we don't say anything about the first order partial derivatives, (a,b) may not be a saddle point.

F **10.** If (a, b) is a critical point of f, then

$$\frac{\partial f}{\partial x}(a, b) = 0 = \frac{\partial f}{\partial y}(a, b).$$

(a,b) could be a point that

$\frac{\partial f}{\partial x}(a,b)$ or $\frac{\partial f}{\partial y}(a,b)$ does NOT exist.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Suppose f'' is continuous on $[1,3]$ and $f(1)=2$, $f(3)=-1$, $f'(1)=2$ and $f'(3)=-1$. Evaluate $\int_1^3 xf''(x)\,dx$. Answer: -2.

Apply integration by parts:

$$\int_1^3 x f''(x)\,dx = \Big[x\cdot f'(x)\Big]_1^3 - \int_1^3 f'(x)\,dx$$

$$= \big(3(-1) - 1\cdot 2\big) - \Big[f(x)\Big]_1^3$$

$$= -5 - \big((-1) - 2\big)$$

$$= -5 + 3$$

$$= -2$$

Ans: $\boxed{-2}$

2. Find the area of the region under the graph of $f(x) = x\ln x$ from $x=1$ to $x=6$. Answer: $18\ln 6 - \frac{35}{4}$.

①
$u = \ln x$, $dv = x\,dx$
$\Rightarrow du = \frac{1}{x}dx$, $v = \frac{1}{2}x^2$

$$\text{Area} = \int_1^6 x\ln x\,dx$$

$$\overset{①}{=} \Big[\tfrac{1}{2}x^2\ln x\Big]_1^6 - \int_1^6 \tfrac{1}{2}x^2\cdot\tfrac{1}{x}\,dx$$

$$= \big(18\ln 6 - 0\big) - \Big[\tfrac{1}{4}x^2\Big]_1^6$$

$$= 18\ln 6 - \tfrac{1}{4}(36-1)$$

$$= 18\ln 6 - \tfrac{35}{4}$$

Ans: $\boxed{18\ln 6 - \frac{35}{4}}$

3. The productivity of a certain country is given by the function

$$f(x, y) = 20x^{3/4}y^{1/4}$$

when x units of labor and y units of capital are used. Find the marginal productivity of labor and the marginal productivity of capital when the amounts expended on labor and capital are 81 units and 16 units respectively.

Answer: Labor $= 10$, Capital $= \frac{135}{8}$

Marginal productivity of labor at $(81, 16)$

$$= \frac{\partial}{\partial x}\left(20x^{\frac{3}{4}}y^{\frac{1}{4}}\right)\Big|_{(81,16)} = 20\cdot\frac{3}{4}x^{-\frac{1}{4}}y^{\frac{1}{4}}\Big|_{(81,16)} = 15\cdot\frac{1}{3}\cdot 2 = 10$$

Marginal productivity of capital at $(81, 16)$

$$= \frac{\partial}{\partial y}\left(20x^{\frac{3}{4}}y^{\frac{1}{4}}\right)\Big|_{(81,16)} = 20\cdot\frac{1}{4}x^{\frac{3}{4}}y^{-\frac{3}{4}}\Big|_{(81,16)} = 5\cdot 27\cdot\frac{1}{8} = \frac{135}{8}$$

4. Use the table of integrals to find $\int e^{2x}\sqrt{5+2e^x}\, dx$.

Table of integrals

(a) $\int u\sqrt{a+bu}\, du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$

(b) $\int \frac{u\, du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$

(c) $\int \frac{du}{1+be^{au}} = u - \frac{1}{a}\ln(1+be^{au}) + C$

Answer: $\frac{1}{15}(3e^x-5)(5+2e^x)^{3/2} + C$

Let $u = e^x$, then $du = e^x dx$

$\Rightarrow \int e^{2x}\sqrt{5+2e^x}\, dx = \int u\sqrt{5+2u}\, du$

Apply (a) with $a=5$, $b=2$ we get

$$\int e^{2x}\sqrt{5+2e^x}\, dx = \frac{1}{30}(6e^x-10)(5+2e^x)^{3/2} + C$$

$$= \frac{1}{15}(3e^x-5)(5+2e^x)^{3/2} + C$$

5. Find the domain of the function $f(x, y) = \dfrac{e^{\sqrt{-x^2-y^2+9}}}{\sqrt{x^2+y^2-4}}$.

Answer: $\{(x,y): 4 < x^2+y^2 \leq 9\}$

The functions in the square roots should be nonnegative and the function in the denominator should be positive.

So $x^2+y^2-4>0$ and $-x^2-y^2+9 \geq 0$.

Therefore, $4 < x^2+y^2 \leq 9$.

The domain is $\{(x,y): 4 < x^2+y^2 \leq 9\}$.

6. Market has two commodities A and B. The demand equations that relate the quantities demanded x and y to the unit prices p and q of the commodities A and B respectively are given by $x = f(p, q) = \dfrac{q}{p+q}$, $y = g(p, q) = e^{-(2q+p^2)}$. Are A and B substitute, complementary or neither? Answer: neither.

$$\frac{\partial f(p,q)}{\partial q} = \frac{(p+q)\cdot 1 - q}{(p+q)^2} = \frac{p}{(p+q)^2} > 0$$

$$\frac{\partial g(p,q)}{\partial p} = -2pe^{-(2q+p^2)} < 0$$

So A and B are neither substitute nor complementary.

7. Find for which p

$$\int_0^\infty e^{-px}\,dx$$

is convergent.

Answer: $p>0$.

For $p>0$, $\int_0^\infty e^{-px}\,dx = \lim_{b\to\infty}\int_0^b e^{-px}\,dx = \lim_{b\to\infty} -\frac{1}{p}e^{-px}\Big|_0^b$

$= \lim_{b\to\infty}\left[\frac{1}{p}-\frac{1}{p}e^{-pb}\right] = \frac{1}{p}$

For $p=0$, $\int_0^\infty e^{-px}\,dx = \lim_{b\to\infty}\int_0^b e^0\,dx = \lim_{b\to\infty} b$ is divergent.

For $p<0$, $\int_0^\infty e^{-px}\,dx = \lim_{b\to\infty}\int_0^b e^{-px}\,dx = \lim_{b\to\infty} -\frac{1}{p}e^{-px}\Big|_0^b$

$= \lim_{b\to\infty}\left[\frac{1}{p}-\frac{1}{p}e^{-pb}\right]$ is divergent

8. Find an equation of the least-squares line for the data (1.2), (2.3), (3.2), (4, 3), (5, 4). Answer:_________.

	x	y	x^2	xy
1	1	2	1	2
2	2	3	4	6
3	3	2	9	6
4	4	3	16	12
5	5	4	25	20
	15	14	55	46

In order to find the least-squares line, we solve the following

$$\begin{cases} 55m+15b=46 & \text{①} \\ 15m+5b=14 & \text{②} \end{cases}$$

① $-3\cdot$ ②

$10m=4$

so $m=\frac{2}{5}$, hence $b=\frac{8}{5}$

Therefore, the least-squares line is $y=\frac{2}{5}x+\frac{8}{5}$

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find each indefinite integral.

a. $\int (\ln x)^2\, dx$

b. $\int \frac{3x}{\sqrt{2x+3}}\, dx$

a. $\int (\ln x)^2 dx \overset{①}{=} x(\ln x)^2 - \int x \cdot 2\ln x \cdot \frac{1}{x}\, dx$

$= x(\ln x)^2 - 2\int \ln x\, dx$

$\overset{②}{=} x(\ln x)^2 - 2\left[x\ln x - \int x \cdot \frac{1}{x} dx\right]$

$= x(\ln x)^2 - 2x\ln x + 2x + C$

$= x((\ln x)^2 - 2\ln x + 2) + C$

① $u = (\ln x)^2,\ dv = dx$
$\Rightarrow du = 2(\ln x)\cdot\frac{1}{x},\ v = x$

② $u = \ln x,\ dv = dx$
$\Rightarrow du = \frac{1}{x}dx,\ v = x$

Ans: $\boxed{x\cdot((\ln x)^2 - 2\ln x + 2) + C}$

b. $\int \frac{3x}{\sqrt{2x+3}} dx \overset{①}{=} 3x(2x+3)^{\frac{1}{2}} - \int 3(2x+3)^{\frac{1}{2}} dx$

$\overset{②}{=} 3x(2x+3)^{\frac{1}{2}} - (2x+3)^{\frac{3}{2}} + C$

① $u = 3x$
$dv = \frac{1}{\sqrt{2x+3}} dx$
$\Rightarrow du = 3dx$
$v = (2x+3)^{\frac{1}{2}}$

Ans: $\boxed{3x(2x+3)^{\frac{1}{2}} - (2x+3)^{\frac{3}{2}} + C}$

(or $(2x+3)^{\frac{1}{2}}\cdot(x-3) + C$)

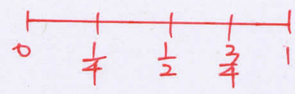
② Let $u = 2x+3$
then $du = 2dx$
$\int 3(2x+3)^{\frac{1}{2}} dx$
$= \int \frac{3}{2} u^{\frac{1}{2}} du$
$= \frac{3}{2}\cdot\frac{2}{3} u^{\frac{3}{2}} + C$
$= (2x+3)^{\frac{3}{2}} + C$

2. (*10* points) Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral

$$\int_0^1 \sqrt{1-x^2}\,dx; \quad n=4.$$

Express each of your answers as a sum of numbers.

Let $f(x)=\sqrt{1-x^2}$: $\Delta x = \frac{1}{4}$



By Trapezoidal Rule

$$\int_0^1 \sqrt{1-x^2}\,dx \approx \frac{\Delta x}{2}\left[f(0)+2\cdot f(\tfrac{1}{4})+2\cdot f(\tfrac{1}{2})+2\cdot f(\tfrac{3}{4})+f(1)\right]$$

$$\approx \frac{1}{8}\left[1+2\cdot\frac{\sqrt{15}}{4}+2\cdot\frac{\sqrt{3}}{2}+2\cdot\frac{\sqrt{7}}{4}+0\right]$$

$$\approx \frac{1}{8}\left[1+\frac{\sqrt{15}}{2}+\sqrt{3}+\frac{\sqrt{7}}{2}\right]$$

$$\boxed{\begin{array}{l}\text{Approx} = \frac{1}{8}\left[1+\frac{\sqrt{15}}{2}+\sqrt{3}+\frac{\sqrt{7}}{2}\right] \\ \left(\text{or } \frac{1}{8}+\frac{\sqrt{15}}{16}+\frac{\sqrt{3}}{8}+\frac{\sqrt{7}}{16}\right)\end{array}}$$

By Simpson's Rule

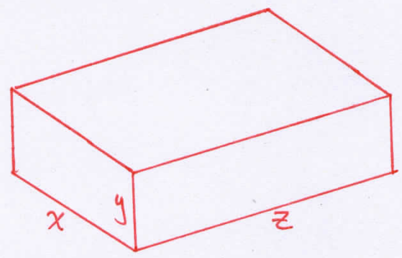
$$\int_0^1 \sqrt{1-x^2}\,dx \approx \frac{\Delta x}{3}\left[f(0)+4f(\tfrac{1}{4})+2f(\tfrac{1}{2})+4f(\tfrac{3}{4})+f(1)\right]$$

$$\approx \frac{1}{12}\left[1+4\cdot\frac{\sqrt{15}}{4}+2\cdot\frac{\sqrt{3}}{2}+4\cdot\frac{\sqrt{7}}{4}+0\right]$$

$$\approx \frac{1}{12}\left[1+\sqrt{15}+\sqrt{3}+\sqrt{7}\right]$$

$$\boxed{\begin{array}{l}\text{Approx} = \frac{1}{12}\left[1+\sqrt{15}+\sqrt{3}+\sqrt{7}\right] \\ \left(\text{or } \frac{1}{12}+\frac{\sqrt{15}}{12}+\frac{\sqrt{3}}{12}+\frac{\sqrt{7}}{12}\right)\end{array}}$$

3. (*10* points) Postal regulations specify that the combined length and girth of a parcel sent by parcel post may not exceed 130 in. Find the dimensions of the rectangular package that would have the greatest possible volume under these regulations.



length + girth = 130

$\Rightarrow z + 2(x+y) = 130$

$\Rightarrow z = 130 - 2x - 2y$

volume $= xyz = xy(130-2x-2y)$

Let $f(x,y) = 130xy - 2x^2y - 2xy^2$, $x \geq 0$, $y \geq 0$, $x+y \leq 65$ ($\because z \geq 0$)

$f_x = 130y - 4xy - 2y^2 = 2y(65-2x-y)$

$f_y = 130x - 2x^2 - 4xy = 2x(65-x-2y)$

$f_x(x,y) = 0 = f_y(x,y) \Rightarrow (x,y)$ is a sol of one of the 4 systems:

① $\begin{cases} x=0 \\ y=0 \end{cases}$ ② $\begin{cases} x=0 \\ 2x+y=65 \end{cases}$ ③ $\begin{cases} y=0 \\ x+2y=65 \end{cases}$ ④ $\begin{cases} x+2y=65 \\ 2x+y=65 \end{cases}$

$\Rightarrow (x,y) = \underbrace{(0,0),\ (0,65),\ (65,0)}_{\text{not interior points of domain of } f},\ \left(\frac{65}{3},\frac{65}{3}\right)$

$f_{xx} = -4y$, $f_{yy} = -4x$, $f_{xy} = 130 - 4x - 4y$.

$D\left(\frac{65}{3},\frac{65}{3}\right) = \left(\frac{-260}{3}\right)\left(\frac{-260}{3}\right) - \left(130 - \frac{260}{3} - \frac{260}{3}\right)^2 > 0$ $f_{xx}\left(\frac{65}{3},\frac{65}{3}\right) = \frac{-260}{3} < 0$

$\Rightarrow f\left(\frac{65}{3},\frac{65}{3}\right)$ is a local maximum, which is also the absolute maximum of f since $\left(\frac{65}{3},\frac{65}{3}\right)$ is the only critical point of f

Also, $z = 130 - \frac{130}{3} - \frac{130}{3} = \frac{130}{3}$

$\Rightarrow$ The optimal dimensions are $\boxed{\frac{65}{3}'' \times \frac{65}{3}'' \times \frac{130}{3}''}$

4. (*10* points) Find the critical point(s) of the function

$$f(x, y) = x^3 + y^3 - x - y.$$

Then use the second derivative test to specify the nature (relative maximum, relative minimum or saddle point) of each point, if possible. Finally, determine the relative extrema of the function.

The domain is $\mathbb{R}$.

First we find all critical points, so we solve $\begin{cases} f_x(x,y) = 3x^2 - 1 = 0 \\ f_y(x,y) = 3y^2 - 1 = 0 \end{cases}$

Then we have $x = \pm\frac{1}{\sqrt{3}}$, $y = \pm\frac{1}{\sqrt{3}}$.

The critical points are $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}), (-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ and $(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$.

$$D(x,y) = f_{xx}(x,y) f_{yy}(x,y) - f_{xy}^2(x,y)$$
$$= 6x \cdot 6y - 0 = 36xy$$

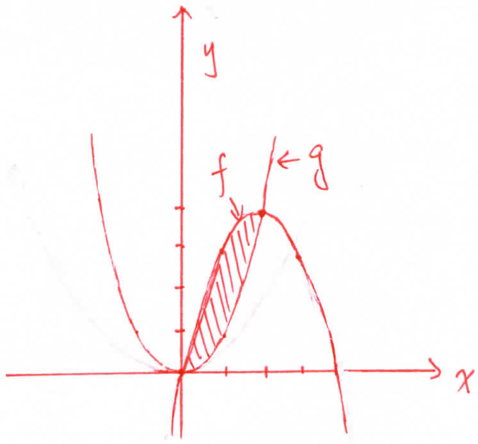
We plug 4 critical points into $D(x,y)$ to obtain $D(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) = 12$, $D(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}) = -12$, $D(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) = -12$ and $D(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}) = 12$.

Since $D(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) > 0$, $f_{xx}(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) > 0$, so f has a relative min $f(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) = -\frac{4}{3\sqrt{3}}$ at $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

$D(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}) > 0$, $f_{xx}(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}) < 0$, so f has a relative max $f(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}) = \frac{4}{3\sqrt{3}}$ at $(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$.

Both $D(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}) < 0$ and $D(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) < 0$, we have $(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$ and $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ are saddle points.

5. (*10* points) Find the volume of the solid obtained by revolving the region bounded by the curves $y = f(x) = -x^2 + 4x$ and $y = g(x) = x^2$ about the x-axis.



First we want to find intersections

$f(x) = -x^2 + 4x = x^2 = g(x)$

$2x^2 - 4x = 0$

namely, $x = 0$ or $x = 2$

So the volume is

$$\int_0^2 \pi (f^2(x) - g^2(x)) \, dx = \int_0^2 \pi [(-x^2 + 4x)^2 - x^4] \, dx$$

$$= \pi \int_0^2 x^4 + 16x^2 - 8x^3 - x^4 \, dx$$

$$= \pi \int_0^2 -8x^3 + 16x^2 \, dx$$

$$= \pi [-2x^4 |_0^2 + \frac{16}{3} x^3 |_0^2]$$

$$= \pi [-32 + \frac{128}{3}]$$

$$= \frac{32}{3} \pi$$

6. (*10* points) It takes x units of labor and y units of capital to produce

$$f(x,y) = 100x^{3/4}y^{1/4}.$$

If a unit of labor costs \$200 and a unit of capital costs \$100 and \$200,000 is budgeted for production, determine how many units should be expended on labor and how many units should be expended on capital in order to maximize production (use the method of Lagrange multipliers ONLY).

Since the budget is 200,000,

$200x + 100y = 200{,}000$,

namely $2x + y = 2{,}000$

we like to maximize $f(x,y) = 100x^{\frac{3}{4}}y^{\frac{1}{4}}$

subject to $g(x,y) = 2x + y - 2{,}000 = 0$,

Let $F(x,y,\lambda) = f(x,y) + \lambda g(x,y)$

$F_x(x,y,\lambda) = 75x^{-\frac{1}{4}}y^{\frac{1}{4}} + 2\lambda = 0$ ①

$F_y(x,y,\lambda) = 25x^{\frac{3}{4}}y^{-\frac{3}{4}} + \lambda = 0$ ②

$F_\lambda(x,y,\lambda) = 2x + y - 2000 = 0$ ③

by ① = 2·② we have

$75\left(\frac{y}{x}\right)^{\frac{1}{4}} + 2\lambda = 50\left(\frac{x}{y}\right)^{\frac{3}{4}} + 2\lambda$

$\frac{y}{x} = \frac{50}{75} = \frac{2}{3}$, so $y = \frac{2}{3}x$.

Together with ③, we obtain

$2x + \frac{2}{3}x = 2000$, so $x = 750$

Hence $y = 500$

Since $f(0, 2{,}000) = f(1{,}000, 0) = 0$,

750 units of labor and 500 units of capital maximize production.

(試題結束)