F1 (工設.

Calculus Test I

2009/07/08

1. The formula $v = 20\sqrt{T}$ provides a good approximation to the speed of sound v in dry air (in m/s) as a function of air temperature T (in kelvins).

10%
(5%×2)

(a) Compute the average rates of change of v with respect to T over the interval $[273, 300]$. What are the units of this rates of change? Draw the corresponding secant line.

(b) Find the instantaneous rates of change when $T = 273k$

2. Find the limit or state that the limit does not exist.

20%
(5%×4)

(a) $\lim\limits_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta}$ (b) $\lim\limits_{h \to 0} \left(\sin h \cdot \cos \dfrac{1}{h}\right)$

(c) $\lim\limits_{x \to 0} x \sin \dfrac{1}{x}$ (d) $\lim\limits_{\theta \to 0} \dfrac{\cos\theta - 1}{\sin\theta}$

3. (a) What is an equation of the tangent line at $x = 3$, assuming that $f(3) = 5$, $f'(3) = 2$

10%
(a) 3%
(b) 7%

(b) Define $f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Show that $f(x)$ is continuous at $x = 0$ but $f'(0)$ does not exist

4. Find $f'(x)$:

20% (5%×4)

(a) $f(x)=\dfrac{\sin x}{x}$ (b) $f(x)=\dfrac{\cos x^2}{1+x^2}$

(c) $f(x)=\sqrt{x+\sqrt{x^2+1}}$ (d) $f(x)=\left(\sqrt{x+1}-1\right)^{\frac{3}{2}}$

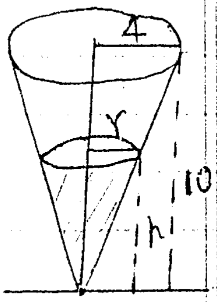
5. Find $\dfrac{dy}{dt}$, where

10% (5%×2)

(a) $\cos(ty)=\dfrac{t^2}{y}$ (b) $y^4-y=t^3+t$

10% (5%×2)

6. Water pours into a conical tank of height 10 ft and radius 4 ft at a rate of 10 ft^3/min.



(a) How fast is the water level rising when it is 5 ft high ?

(b) As time passes, what happens to the rate at which the water level rises ? Explain.

7. Suppose $f(x)$ satisfies : $f''(x)=-f(x)$ ——— $(*)$

10% (5%×2)

(a) Show that $\left(f(x)\right)^2+\left(f'(x)\right)^2=\left(f(0)\right)^2+\left(f'(0)\right)^2$

(b) Verify that $\cos x$ and $\sin x$ satisfy $(*)$ and deduce that $\sin^2 x+\cos^2 x=1$

8. Define $g(x)=\begin{cases} x^2\sin\frac{1}{x} & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$

10%

Use the limit definition to show : $g'(0)$ exists but $g'(0)\neq\lim\limits_{x\to 0}g'(x)$.

國立中央大學考試試卷

National Central University Examination Answer Sheet

☐ 平時考(Quiz) ☐ 期中考(Midterm) ☐ 期末考(Final)		評分 (Score)
科目 (Course Title)	日期 (Date) 7/8	
系/級 (Department / Grade)	班/組 (Class)	
學號 (Student ID)	姓名 (Name)	

1. 參考 §2.1 Example 2.

2. (a) $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta} = \lim_{\theta\to 0} \frac{\cos^2\theta - 1}{\theta(\cos\theta+1)} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta}\cdot\frac{\sin\theta}{\cos\theta+1} = 0.$

(b) $\because -1 \le \cos\frac{1}{h} \le 1 \Rightarrow -\sin h \le \sin h\cdot\cos\frac{1}{h} \le \sin h$ &

$\lim_{h\to 0} -\sin h = 0 = \lim_{h\to 0} \sin h$

By Squeeze Thm., $\lim_{h\to 0} \sin h\cdot\cos\frac{1}{h} = 0$

(c) 參考 §2.6 Example 1.

(d) $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta\to 0} \frac{\cos^2\theta - 1}{\sin\theta(\cos\theta+1)} = \lim_{\theta\to 0} \frac{\sin\theta}{\cos\theta+1} = 0$

3. (a) Let an equation of tangent line be $y = mx + b$.

$\because f(3) = 5$, $f'(3) = 2$

$\therefore m = 2$ and $b = -1$

Hence, $y = 2x - 1$

(b) 1° $\because \lim_{x\to 0} f(x) = \lim_{x\to 0} x\cdot\sin\frac{1}{x} = 0 = f(0)$

$\therefore f(x)$ is continuous at $x = 0$.

2° $f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h\cdot\sin\frac{1}{h}}{h} = \lim_{h\to 0} \sin h$

$\therefore f'(0)$ does not exist.

4. (a) $f'(x) = \frac{\cos x \cdot x - \sin x}{x^2} = \frac{1}{x}\cos x - \frac{1}{x^2}\sin x$

(b) $f'(x) = \frac{2x\cdot(-\sin x^2)\cdot(1+x^2) - 2x\cdot\cos x^2}{(1+x^2)^2} = -\frac{2x}{1+x^2}\sin x^2 - \frac{2x}{(1+x^2)^2}\cos x^2$

(c) 參考 §3.7 Example 6.

(d) $f'(x) = \frac{3}{2}\left(\sqrt{x+1}-1\right)^{1/2}\cdot\frac{1}{2}\frac{1}{\sqrt{x+1}}$

5. (a) 參考 §3.8 Example 3.

(b) $\frac{d}{dt}(y^4 - y) = \frac{d}{dt}(t^3+t)$

$\Rightarrow 4y^3\cdot\frac{dy}{dt} - \frac{dy}{dt} = 3t^2+1$

$\Rightarrow \frac{dy}{dt} = \frac{3t^2+1}{4y^3-1}$

6. 參考 §3.9 Example 3.

7. (a) Let $g(x) = f^2(x) + f'^2(x)$

$\Rightarrow g'(x) = 2\cdot f'(x)\cdot f(x) + 2\cdot f''(x)\cdot f'(x)$

$\because f''(x) = -f(x) \Rightarrow g'(x) = 0$

$\therefore g(x)$ is constant for all x, we say $g(x) = C$

$C = g(0) = f^2(0) + f'^2(0)$

Hence, $f^2(x) + f'^2(x) = f^2(0) + f'^2(0)$.

(b) 1° $(\cos x)'' = -\cos x$ and $(\sin x)'' = -\sin x$

2° let $f(x) = \sin x \Rightarrow f''(x) = -\sin x = -f(x)$ and $f'(x) = \cos x$

by (a), we have $\sin^2 x + \cos^2 x = 1$.

8. 1° $g'(0) = \lim_{h\to 0} \frac{g(0+h)-g(0)}{h}$

$= \lim_{h\to 0} \frac{h^2 \sin\frac{1}{h}}{h}$

$= \lim_{h\to 0} h\cdot \sin\frac{1}{h} = 0.$

Hence, $g'(0)$ exists and $g'(0)=0$.

2° For $x \neq 0$, $g'(x) = 2x\cdot\sin\frac{1}{x} - \cos\frac{1}{x}$

$\Rightarrow \lim_{x\to 0} g'(x)$ does not exist

Hence, $g'(0) \neq \lim_{x\to 0} g'(x)$.

2

Test II

2009/07/15 P1

1 Find the maximum and minimum of $f(x)$ on I:

15% (5%×3)

(i) $f(x) = \dfrac{x}{x^2+5}$, $I = [-4, 8]$

(ii) $f(x) = \sqrt{x+x^2} - 2x$, $I = [0, 4]$

(iii) $f(x) = \cos^2 x + \sin x$, $I = [0, \pi]$

2. Find the critical points, local maximum, local minimum, the points of inflection of $f(x)$ if any one them exists. Furthermore sketch the graph of $f(x)$.

20% (5%×4)

(i) $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$

(ii) $f(x) = \cos^2 x + \sin x$

(iii) $f(x) = \dfrac{2x+1}{x^2+1}$ (iv) $f(x) = \dfrac{3x+2}{2x-4}$

3. The volume of a right circular cone is $\frac{\pi}{3} r^2 h$ and its area is $S = \pi r \sqrt{r^2+h^2}$. Find the dimensions of the cone with surface area 1 and maximal volume.

10%

4. Find the following indefinite integral:

20% (4%×5)

(i) $\int (3x^4 - 5x^{\frac{2}{3}} + x^{-3} + 5)\, dx$

(ii) $\int (\sin(2t-9) + 2.0 \cos 3t)\, dt$

(iii) $\int (4t-9)^{-3}\, dt$

(iv) $\int (9t^{-\frac{2}{3}} + 4t^{\frac{7}{3}})\, dt$

(v) $\int \dfrac{x^2+2x-3}{x^4}\, dx$

5. Solve the equation to get $y(t)$:

15% (5%×3)

(i) $\frac{dy}{dt} = \sin(\pi t)$, $y(2) = 2$

(ii) $\frac{dy}{dt} = \cos 5t$, $y(\pi) = 3$

(iii) $y''(t) = t - \cos t$, $y'(0) = 2$, $y(0) = -2$

6. Estimate the fallowing quantity using the Linear Approximation :

10% (5%×2)

(a) $(16.5)^{\frac{1}{4}} - 16^{\frac{1}{4}}$ (b) $(15.8)^{\frac{1}{4}}$

7. Design a cylindrical can of volume $10\ ft^3$ so that it uses the least amount of metal. In other words, minimize the surface area of the can.

10%

國立中央大學考試試卷

National Central University Examination Answer Sheet

☑ 平時考(Quiz) ☐ 期中考(Midterm) ☐ 期末考(Final)		評分 (Score)
科目 (Course Title)	日期 (Date) 7/15	
系/級 (Department / Grade)	班/組 (Class)	
學號 (Student ID)	姓名 (Name)	

1.

(i) $f'(x) = \frac{5-x^2}{(x^2+5)^2}$

$\Rightarrow$ critical pts. : $x = \pm\sqrt{5}$

$f(-4) = -\frac{4}{21}$, $f(-\sqrt{5}) = -\frac{\sqrt{5}}{10}$, $f(\sqrt{5}) = \frac{\sqrt{5}}{10}$, $f(8) = \frac{8}{69}$

Hence, $f(\sqrt{5}) = \frac{\sqrt{5}}{10}$ is the maximum on $[-4, 8]$

$f(-\sqrt{5}) = -\frac{\sqrt{5}}{10}$ is the minimum on $[-4, 8]$

(ii) $f'(x) = \frac{1+2x-4\sqrt{x(1+x)}}{2\sqrt{x(1+x)}}$

$\Rightarrow$ critical pts. : $x = 0, -\frac{1}{2}+\frac{1}{\sqrt{3}}$

$f(0) = 0$, $f(-\frac{1}{2}+\frac{1}{\sqrt{3}}) = 1-\frac{\sqrt{3}}{2}$, $f(4) = 2\sqrt{5}-8$

Hence, $f(-\frac{1}{2}+\frac{1}{\sqrt{3}}) = 1-\frac{\sqrt{3}}{2}$ is the maximum on $[0,4]$

$f(4) = 2\sqrt{5}-8$ is the minimum on $[0,4]$

(iii) 參考 §4.3 Example 4.

2.

(i) $f'(x) = 12x\cdot(x-1)^2 \Rightarrow$ critical pts: $x = 0, 1$

$f''(x) = 12(x-1)(3x-1)$

x	$(-\infty,0)$	0	$(0,\frac{1}{3})$	$\frac{1}{3}$	$(\frac{1}{3},1)$	1	$(1,\infty)$
$f'(x)$	−	0	+	+	+	0	+
$f''(x)$	+	+	+	0	−	0	+

$f''=0$ & change sign

a point of inflection of $f(x)$: $x = \frac{1}{3}, 1$

local min.: $f(0) = 1$

local max.: none.

圖型參考 P.210

(ii) 參考期中考 2. (ii)

(iii) $f'(x) = \frac{-2(x^2+x-1)}{(x^2+1)^2} \Rightarrow$ critical pts: $x = \frac{-1\pm\sqrt{5}}{2}$

$f''(x) = \frac{4x^5+6x^4-8x^3+4x^2-12x-2}{(x^2+1)^4} = \frac{(x^2+1)(4x^3+6x^2-12x-2)}{(x^2+1)^4}$

x	$(-\infty,\frac{-1-\sqrt{5}}{2})$	$\frac{-1-\sqrt{5}}{2}$	$(\frac{-1-\sqrt{5}}{2},\frac{-1+\sqrt{5}}{2})$	$\frac{-1+\sqrt{5}}{2}$	$(\frac{-1+\sqrt{5}}{2},\infty)$
$f'(x)$	−	0	+	0	−

note: $\lim_{x\to\pm\infty} f(x) = 0$

$\lim_{x\to\pm\infty} f'(x) = 0$

local min.: $f(\frac{-1-\sqrt{5}}{2}) = \frac{-2\sqrt{5}}{5+\sqrt{5}}$

local max.: $f(\frac{-1+\sqrt{5}}{2}) = \frac{2\sqrt{5}}{5-\sqrt{5}}$

(iv) $f(x) = \frac{3x+2}{2x-4}$ is not defined at $x=2$

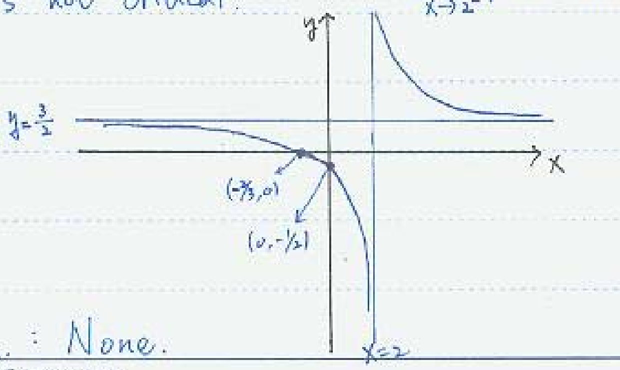
note: $\lim_{x\to 2^+} f(x) = +\infty$

$\lim_{x\to 2^-} f(x) = -\infty$

$f'(x) = -\frac{4}{(x-2)^2} \Rightarrow x=2$ is not critical.

$f''(x) = \frac{8}{(x-2)^3}$

x	$(-\infty,2)$	$(2,\infty)$
$f'(x)$	−	−
$f''(x)$	−	+



local min. & local max.: None.

3.

$S = \pi r\sqrt{r^2+h^2} = 1 \Rightarrow h^2 = \frac{1-\pi^2 r^4}{\pi^2 r^2}$

$r=0 \Rightarrow S\neq 1$

$\Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{1}{3} r\sqrt{1-\pi^2 r^4}\ ,\quad 0<r\leq \frac{1}{\sqrt{\pi}}$

1° $\frac{d}{dr}V(r) = \frac{1}{3}\sqrt{1-\pi^2 r^4} + \frac{r}{6}\frac{-4\pi^2 r^3}{\sqrt{1-\pi^2 r^4}}$

$\frac{d}{dr}V(r) = 0 \Rightarrow r = (3\pi^2)^{-\frac{1}{4}}$

2° $\because h^2 = \frac{1-\pi^2 r^4}{\pi^2 r^2} \Rightarrow h = \frac{\sqrt{2}}{3^{1/4}\sqrt{\pi}}$

Since, $\lim_{r\to 0^+} V(r) = 0$ and $V(\frac{1}{\sqrt{\pi}}) = 0$

$V((3\pi^2)^{-1/4}) = \frac{1}{3^{3/4}}\sqrt{\frac{2}{\pi}}$

Hence, $r = \frac{1}{3^{1/4}\sqrt{\pi}}$, $h = \frac{\sqrt{2}}{3^{1/4}\sqrt{\pi}}$

4.

(i) $\frac{3}{5}x^5 - 3x^{\frac{5}{3}} - \frac{1}{2}x^{-2} + 5x + C$

(ii) 參考 §4.8 Example 3.

(iii) $-\frac{1}{8}(4t-9)^{-2} + C$

(iv) $27t^{\frac{1}{3}} + \frac{6}{5}t^{\frac{10}{3}} + C$

(v) $-x^{-1} - x^{-2} + x^{-3} + C$

5.

(i) 參考 §4.8 Example 5.

(ii) $y(t) = \frac{1}{5}\sin 5t + 3$

(iii) $y(t) = \frac{1}{6}t^3 + \cos t + 2t - 3$

7. 參考 §4.6 Example 4.

6. $\Delta f = f(a+\Delta x) - f(a) \approx f'(a)\cdot\Delta x$

(a) let $f(x) = x^{\frac{1}{4}}$, $a=16$, $\Delta x = 0.5$ $\Rightarrow (16.5)^{1/4} - (16)^{1/4} \approx f'(16)\times 0.5 = 1/64$

(b) let $f(x) = x^{\frac{1}{4}}$, $a=16$, $\Delta x = -0.2$ $\Rightarrow (15.8)^{1/4} \approx (16)^{1/4} + \Delta f = 2 - \frac{1}{160}$

1 Calculate the follow sums : $\sum_{j=3}^{4} \sin(j\frac{\pi}{2})$

2. $\lim_{N\to\infty} \frac{\pi}{2N} \sum_{j=1}^{N} \sin(\frac{\pi}{3}+\frac{j\pi}{2N})$

3 Evalute $\lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \sqrt{1-(\frac{j}{N})^2}$ by interpreting it as the area of part of a family geometric figure.

4 Prove that $0.277 \le \int_{\pi/8}^{\pi/4} \cos x \, dx \le 0.363$

5 $\int_{\pi/4}^{\pi/2} \frac{\sin x}{x} dx \le \frac{\sqrt{2}}{2}$

6 $\int_0^5 |x^2-4x+3| dx$

7 What is the area (a positive number) between the x-axis and the graph of $f(x)$ on $[1.3]$ if $f(x)$ is a negative function whose antiderivative F has the values $F(1)=7$ and $F(3)=4$

8 $\frac{d}{dx} \int_0^{x^2} \sin^2 t \, dt$

9 $\frac{d}{dx} \int_{x^2}^{x^4} \sqrt{t} \, dt$

10 Show that $\int_0^{\pi/6} f(\sin\theta) d\theta = \int_0^{1/2} f(u) \frac{1}{\sqrt{1-u^2}} du$

11 Sketch a region whose area is represented by $\int_{-\sqrt{2}/2}^{\sqrt{2}/2} (\sqrt{1-x^2} - |x|) dx$ and evaluate using geometry

1. Find the local extreme values in the domain $\{x : x > 0\}$ and use the Second Derivative Test to determine whether these values are local maxima or minima.

§7.3
#57

$$f(x) = \frac{\ln x}{x}$$

$f'(x) = \frac{1-\ln x}{x^2} = 0$ when $1 = \ln x$, $x = e$

$f''(x) = \frac{-3+2\ln x}{x^3}$, $f''(e) = \frac{-1}{e^3} < 0$

$\therefore f(e)$ is a local maximum

2. Show that for any x, $\lim\limits_{n\to\infty}\left(1+\frac{x}{n}\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{nx}$.

§7.5
#25

Hint: $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

let $t = \frac{n}{x}$, 左式 $= \underbrace{\lim\limits_{t\to\infty}\left(1+\frac{1}{t}\right)^{tx}}_{= e^x} =$ 右式

3. Apply L'Hôptial's Rule to evalate the limit.

(a) $\lim\limits_{x\to 0}\left(\cot x - \frac{1}{x}\right)$

§7.7
#21

(b) $\lim\limits_{x\to 0} x^{\sin x}$

§7.7
#40

(c) $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^x$

§7.7
#48

Hint:

(a) $\cot x - \frac{1}{x} = \frac{x\cdot\cos x - \sin x}{x\cdot\sin x}$

(b) $x^{\sin x} = e^{\sin x\cdot(\ln x)}$

$\sin x\cdot\ln x = \frac{\ln x}{\frac{1}{\sin x}}$

(c) $\left(\frac{x}{x+1}\right)^x = e^{x\cdot\ln\left(\frac{x}{x+1}\right)}$

$x\cdot\ln\left(\frac{x}{x+1}\right) = \frac{\ln\left(\frac{x}{x+1}\right)}{\frac{1}{x}}$

4. Find the derivative.

(a) $y = e^{\cos^{-1}x}$

§7.8
#34

(b) $y = \ln(\sin^{-1}t)$

§7.8
#46

(a) $y' = e^{\cos^{-1}x}\frac{d}{dx}\cos^{-1}x$

$= e^{\cos^{-1}x}\frac{-1}{\sqrt{1-x^2}}$

(b) $y' = \frac{1}{\sin^{-1}t}\frac{d}{dt}\sin^{-1}t$

$= \frac{1}{\sin^{-1}t}\frac{1}{\sqrt{1-t^2}}$

5. Calculate the (indefinite) integral.

(a) $\int \frac{dx}{x\ln x}$

§7.3
#59

(a) let $u = \ln x \Rightarrow du = \frac{1}{x}dx$

原式 $= \int \frac{1}{u}\,du = \ln|u| + C$

(b) $\int \frac{e^x\,dx}{1+e^{2x}}$

§7.8
#77

(b) let $u = e^x \Rightarrow du = e^x\,dx$

原式 $= \int \frac{1}{1+u^2}\,du = \tan^{-1}u + C$

(c) $\int \sinh^2 x\cdot\cosh x\,dx$

§7.9
#34

(c) let $u = \sinh x \Rightarrow du = \cosh x\,dx$

原式 $= \int u^2\,du = \frac{1}{3}u^3 + C$

(d) $\int e^{-x}\cdot\sinh x\,dx$

§7.9
#41

(d) 原式 $= \frac{1}{2}\int e^{-x}(e^x - e^{-x})\,dx$

$= \frac{1}{2}\int 1 - e^{-2x}\,dx$

$= \frac{1}{2}x + \frac{1}{4}e^{-2x} + C$

Note:

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

(e) $\int x^{-9} \cdot \ln x \, dx$

§8.2 #17

(e) 原式 $= (\ln x)\left(-\frac{1}{8}x^{-8}\right) - \int \left(-\frac{1}{8}x^{-8}\right)\cdot\frac{1}{x}\,dx$

$= -\frac{\ln x}{8x^8} + \frac{1}{8}\int x^{-9}\,dx$

(f) $\int x \cdot \cosh 2x \, dx$

§8.2 #29

(f) 原式 $= x\cdot\frac{1}{2}\sinh 2x - \int \frac{1}{2}\sinh 2x\,dx$ $\quad u = x,\ v = \frac{1}{2}\sinh 2x$

$= \frac{x}{2}\sinh 2x - \frac{1}{2}\int \sinh 2x\,dx = \frac{x}{2}\sinh 2x - \frac{1}{4}\cosh 2x + C$

(g) $\int \frac{\ln(\ln x)}{x}\,dx$

§8.2 #36

(g) let $u = \ln x \Rightarrow du = \frac{1}{x}dx$

原式 $= \int \ln u\,du = u\cdot\ln u - \int u\cdot\frac{1}{u}\,du = u\ln u - u + C$

(h) $\int \cos^3(\pi\theta)\cdot\sin^4(\pi\theta)\,d\theta$

§8.3 #25

(h) let $u = \pi\theta$, 原式 $= \frac{1}{\pi}\int \cos^3 u\,\sin^4 u\,du$

$= \frac{1}{\pi}\int (1-\sin^2 u)\cdot\sin^4 u\cdot\cos u\,du$

(let $v = \sin u \Rightarrow dv = \cos u\,du$)

$= \frac{1}{\pi}\int (1-v^2)v^4\,dv = \frac{1}{\pi}\int v^4 - v^6\,dv$

(i) $\int \frac{\tan^3(\ln t)}{t}\,dt$

§8.3 #3

(i) let $u = \ln t$

原式 $= \int \tan^3 u\,du = \int \tan u\,(\sec^2 u - 1)\,du$

$= \int \tan u\cdot\sec^2 u\,du - \int \tan u\,du$

let $v = \tan u$ $\qquad = \ln|\sec u| + C_2$

$= \int \frac{u}{v}\,d\frac{u}{v} = \frac{1}{2}\frac{u^2}{v} + C_1$

(j) $\int \frac{dx}{\sqrt{25+x^2}}$

§8.4 #22

(j) let $x = 5\cdot\tan\theta \Rightarrow dx = 5\cdot\sec^2\theta\,d\theta$, $25 + x^2 = 25\sec^2\theta$

原式 $= \int \frac{5\sec^2\theta}{5\cdot\sec\theta}\,d\theta = \ln|\sec\theta + \tan\theta| + C$

$\sec\theta = \sqrt{25+x^2}/5$

$\tan\theta = x/5$

(k) $\int \frac{(x^2+11x)\,dx}{(x-1)(x+1)^2}$

§8.5 #17

(k) $\frac{x^2+11x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\Rightarrow A = 3,\ C = 5,\ B = -2$

原式 $= 3\int \frac{1}{x-1}dx - 2\int\frac{1}{x+1}dx + 5\int\frac{1}{(x+1)^2}dx$

$= 3\cdot\ln|x-1| - 2\ln|x+1| - \frac{5}{x+1} + C$

(l) $\int_{-\infty}^{0} x^2 e^{-x}\,dx$

§8.6 #32

(l) $\because \int x^2 e^{-x}\,dx \overset{\text{integration by parts}}{=} -e^{-x}(x+1)^2 + C$

$\therefore \int_R^0 x^2 e^{-x}\,dx = -e^{-x}(x+1)^2\Big|_R^0 = -1 + e^{-R}(R+1)^2$

Hence. $\int_{-\infty}^{0} x^2 e^{-x}\,dx = \lim_{R\to-\infty}\left(-1+e^{-R}(R+1)^2\right) = \infty$.

(m) $\int e^t\sqrt{e^t+1}\,dt$

§7.1 #61

(m) let $u = e^t + 1$. 原式 $= \int\sqrt{u}\,du = \frac{2}{3}u^{3/2} + C$

6. Set $I_m = \int_0^{\pi/2} \sin^m x\,dx$.

§8.3 #75

(a) Show that $I_1 = 1$, $I_2 = \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)$ and prove that for $m > 1$, $I_m = \left(\frac{m-1}{m}\right)I_{m-2}$

$I_m = \int_0^{\pi/2}\sin^m x\,dx$

$= -\frac{\sin^{m-1}x\cdot\cos x}{m}\Big|_0^{\pi/2} + \frac{m-1}{m}\int_0^{\pi/2}\sin^{m-2}x\,dx$

(b) Show that $I_3 = \frac{2}{3}$ and $I_4 = \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)$.

$I_3 = \frac{3-1}{3}I_1 = \frac{2}{3}$ $\qquad I_4 = \frac{4-1}{4}I_2 = \frac{3}{4}I_2 = \frac{3}{4}\,\frac{1}{2}\,I_0 = \frac{3}{4}\,\frac{1}{2}\,\frac{\pi}{2}$

(c) Show more generally:

$$I_{2m} = \frac{2m-1}{2m}\,\frac{2m-3}{2m-2}\cdots\frac{1}{2}\,\frac{\pi}{2}$$

$$I_{2m+1} = \frac{2m}{2m+1}\,\frac{2m-2}{2m-1}\cdots\frac{2}{3}$$

(c) I_{2m}

① $m = 1, 2$ it is true

② $m = k-1$: $I_{2(k-1)} = I_{2k-2} = \frac{2k-3}{2k-2}\,\frac{2k-5}{2k-4}\cdots\frac{1}{2}\,\frac{\pi}{2}$

$I_{2k} = \left(\frac{2k-1}{2k}\right)I_{2k-2} = \frac{2k-1}{2k}\,\frac{2k-3}{2k-2}\cdots\frac{1}{2}\,\frac{\pi}{2}$

by induction.

I_{2m+1}

By the same way!