DIFFERENTIAL EQUATION MA2041-B

THE FINAL EXAM

1. Determine whether the given differential equation is exact. If it is exact, solve it.

$$(x - y^3 + y^2 \sin x)dx = (3xy^2 + 2y \cos x)dy$$

2. Solve the given equation by using an appropriate substitution.

$$x\frac{dy}{dx} + y = x^2y^2$$

3. Solve the given boundary-value problem.

$$y^{''} + y = 0, \quad y(0) = 1, y(\pi/2) = 0.$$

4. Solve the given differential equation by undetermined coefficients (Annihilator approach).

$$y^{''} + 6y^{'} + 8y = 3e^{-2x} + 2x$$

5. Find power series solutions of the given differential equation about the point $x = 0$

$$(x^2 + 2)y^{''} + xy' - 3y = 0$$

6. Find series solutions about $x = 0$

$$2xy^{''} + 5y^{'} + xy = 0$$

7. Find the general solution of the given differential equation on $(0, \infty)$.

$$\frac{d}{dx}[xy^{'}] + \big(x - \frac{4}{x}\big)y = 0$$

8. Using the indicated change of variable to find the general solution of the given differential equation on $(0, \infty)$.

$$x^2y^{''} + 2xy^{'} + \alpha^2x^2y = 0; \quad y = x^{-1/2}\nu(x)$$

9. Using Laplace transform, solve the initial-value problem.

$$y^{''} - 3y^{'} + 2y = e^{-4t}, \quad y(0) = 4, \quad y^{'}(0) = 2$$

10. Using Laplace transform, solve the initial-value problem.

$$y^{''} - 6y^{'} + 9y = t^2e^{3t}, \quad y(0) = 1, \quad y^{'}(0) = 3$$