MIDTERM EXAM

1. Verify that the indicated function $y = x + 4\sqrt{x+2}$ is an explicit solution of the given first-order differential equation

$$(y-x)y' = y-x+8$$

2. Let P(t) denote the total population at time t. Assume that the rate at which the population grows at a certain time is proportional to the total population at that time. The assumption can be expressed as

$$\frac{dP}{dt} = kP$$

where k is a constant of proportionality.

Determine a differential equation for the population P(t) of a country when people are allowed to immigrate into the country at a constant rate i > 0. What is the differential equation for the population P(t) of the country when individuals are allowed to emigrate from the country at a constant rate e > 0? What is the equation for the population P(t) of the country when people can immigrate and emigrate at the same time?

3. Determine whether the given differential equation is exact. If it is exact, solve it.

$$(x - y^3 + y^2 \sin x)dx = (3xy^2 + 2y \cos x)dy$$

4. Solve the given equation by using an appropriate substitution.

$$x\frac{dy}{dx} + y = x^2y^2$$

5. Find an explicit solution of the given initial-value problem

$$\frac{dy}{dx} = \frac{x^2 - 1}{y^2 - 1}, \quad y(2) = 2$$

6. Solve the given initial-value problem

$$\frac{dy}{dx} + y = f(x), \quad y(0) = 0$$

where

$$f(x) = \begin{cases} 1, & 0 \le x \le 3\\ 0, & x > 3 \end{cases}$$

7. Find the general solution of the given differential equation

$$x\frac{dy}{dx} + 4y = x^3 - x$$

8. Find the value of k so that the given equation is exact, and solve the respective equation.

$$(y^3 + kxy^4 - 2x)dx + (3xy^2 + 20x^2y^3)dy = 0$$

9. Solve the given differential equation by using appropriate substitution.

$$\frac{dy}{dx} = (x+y+1)^2$$

10. Solve the given differential equation

$$(y^2 + yx)dx - x^2dy = 0$$