Calculus II : Final Exam 2009/09/09

1. Find the solution $y(t)$:

(a) $y' + \frac{1}{t+1} y = t^{-2}$, $y(1) = 2$

(b) $y'' + 2y' - 8y = 0$, $y(0) = 2$, $y'(0) = 2$

(c) Find a power series solution of the following equation.

$$\begin{cases} y'' - ty' + y = 0 \\ y(0) = 1, \quad y'(0) = 0 \end{cases}$$

2. Whether the infinite series is convergent or divergent? why?

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!}$

(c) $\sum_{n=1}^{\infty} \sin\frac{1}{n}$ (d) $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

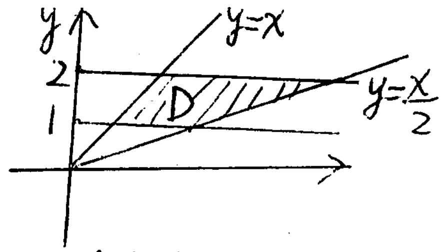
(e) $\sum_{n=1}^{\infty} \frac{1}{n \cdot \ln n}$ (f) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^4}{n^3 + 1}$

3. Find the range and radius of convergence:

(a) $\sum_{n=1}^{\infty} \frac{x^n}{(n!)^2}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+1}}$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

4. (a) Find $\iint_D \frac{\sin y}{y}\, dA$, where D:



(b) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$

5. (a) Suppose that $x = g(t,s)$, $y = h(t,s)$. Show that

$$(*)\quad f_{tt} = f_{xx}\left(\frac{\partial x}{\partial t}\right)^2 + 2 f_{xy}\left(\frac{\partial x}{\partial t}\right)\left(\frac{\partial y}{\partial t}\right) + f_{yy}\left(\frac{\partial y}{\partial t}\right)^2 + f_x \frac{\partial^2 x}{\partial t^2} + f_y \frac{\partial^2 y}{\partial t^2}$$

(b) Use above equation (*) to prove that in polar coordinates (r, θ),

$$\Delta f = f_{xx} + f_{yy} = f_{rr} + \frac{1}{r^2} f_{\theta\theta} + \frac{1}{r} f_r$$

6. (a) By investing x units of labor and y unit of capital, a low-end watch manufacturer can product $P(x,y) = 50x^{0.4}y^{0.6}$ watches.
Find the maximum number of watches that can be produced on a budget of \$20,000 if labor cost \$100 per unit and capital costs \$200 per unit.

(b) Find the local maximum or minimum if it exists of $f(x,y) = y^2x - yx^2 + xy$. Give your reason.

7. Evaluate $\iint_D (x+y)\, dx\, dy$, where $D = \{(x,y) \mid x^2+y^2 \leq 4,\ y \geq 0\}$.