Calculus Homework Assignment 4

Class: ______________

Student Number: ______________

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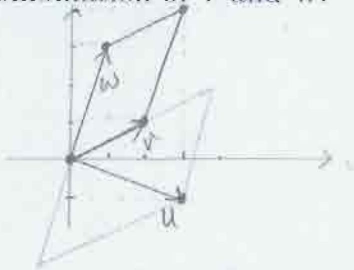
1. Sketch the parallelogram spanned by $\mathbf{v} = \langle 2, 1\rangle$ and $\mathbf{w} = \langle 1, 3\rangle$. Add the vector $\mathbf{u} = \langle 3, -1\rangle$ to the sketch and express $\mathbf{u}$ as a linear combination of $\mathbf{v}$ and $\mathbf{w}$. [like §13.1 #57]



$\vec{u} = a\vec{v} + b\vec{w}$.

$\langle 3, -1\rangle = a\langle 2, 1\rangle + b\langle 1, 3\rangle$

$= \langle 2a+b, a+3b\rangle$

$a = 2$

$b = -1$.

2. Find parametric equations of the line with the given description.

a. Passes through $(1, 1, 1)$ and $(3, -5, 2)$

b. Perpendicular to the yz−plane passing through the point $(0, 0, 2)$ [§13.2 #31.37]

a. $\begin{cases} x = 1 + 2t \\ y = 1 - 6t \\ z = 1 + t \end{cases}$

b. $\begin{cases} x = t \\ y = 0 \\ z = 2 \end{cases}$

3. Determine if the lines $\mathbf{r}_1(t) = \langle 2, 1, 1\rangle + t\langle -4, 0, 1\rangle$ and $\mathbf{r}_2(s) = \langle -4, 1, 5\rangle + s\langle 2, 1, -2\rangle$ intersect and, if so, find the point of intersection. [like §13.2 #51]

$r_1(t) = r_2(s)$

$\Rightarrow (*) \begin{cases} x = 2 - 4t = -4 + 2s & — ① \\ y = 1 = 1 + s & — ② \\ z = 1 + t = 5 - 2s & — ③ \end{cases}$

② ⇒ $s = 0$ 代入①

⇒ $t = \frac{3}{2}$ 代入③

⇒ $s = \frac{5}{4} \neq 0$

⇒ (*) has no solution.

Hence $r_1(t)$ and $r_2(t)$ don't intersect.

4. Find the decomposition $\mathbf{a} = \mathbf{a}_{\parallel} + \mathbf{a}_{\perp}$ with respect to $\mathbf{b}$ where $\mathbf{a} = \langle 3, 5, 5\rangle$ and $\mathbf{b} = \langle 5, 2, 2\rangle$. [§13.3 #54]

$\vec{a} \cdot \vec{b} = \langle 3, 5, 5\rangle \cdot \langle 5, 2, 2\rangle = 35$

$\vec{b} \cdot \vec{b} = \langle 5, 2, 2\rangle \cdot \langle 5, 2, 2\rangle = 33$.

$\vec{a}_{\parallel} = \left(\frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\right)\vec{b}$

$= \frac{35}{33}\langle 5, 2, 2\rangle = \langle \frac{175}{33}, \frac{70}{33}, \frac{70}{33}\rangle$

$\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel}$

$= \langle 3, 5, 5\rangle - \langle \frac{175}{33}, \frac{70}{33}, \frac{70}{33}\rangle$

$= \langle \frac{-76}{33}, \frac{95}{33}, \frac{95}{33}\rangle$

$\Rightarrow \vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$

$= \langle \frac{175}{33}, \frac{70}{33}, \frac{70}{33}\rangle + \langle \frac{-76}{33}, \frac{95}{33}, \frac{95}{33}\rangle$

5. What is the angle between $\mathbf{v}$ and $\mathbf{w}$ if:

a. $\mathbf{v}\cdot\mathbf{w} = -\|\mathbf{v}\|\|\mathbf{w}\|$

b. $\mathbf{v}\cdot\mathbf{w} = \frac{1}{2}\|\mathbf{v}\|\|\mathbf{w}\|$ [§13.3 #58]

a. $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}$

$= \dfrac{-\|\vec{v}\|\|\vec{w}\|}{\|\vec{v}\|\|\vec{w}\|} = -1$

$\Rightarrow \theta = 180°$

b. $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}$

$= \dfrac{\frac{1}{2}\|\vec{v}\|\|\vec{w}\|}{\|\vec{v}\|\|\vec{w}\|} = \frac{1}{2}$

$\Rightarrow \theta = 60°$

7. Find a vector X such that

$$\langle 1,1,1\rangle \times X = \langle 1,-1,0\rangle.$$

[§13.4 #61]

Let $X = \langle a, b, c\rangle$.

$\langle 1,1,1\rangle \times \langle a,b,c\rangle = \langle 1,-1,0\rangle$

$\Rightarrow \begin{cases} c-b=1 \\ a-c=-1 \\ b-a=0 \end{cases} \Rightarrow \begin{matrix} b=a \\ c=a+1 \end{matrix}$

$\Rightarrow X = \langle a, a, a+1\rangle$

Take $a=1$

Then $X = \langle 1, 1, 2\rangle$

6. Use the cross product to find the area of the triangle with vertices $P=(1,1,5)$, $Q=(3,4,3)$, and $R=(1,5,7)$. [§13.4 #48]

$\vec{PQ} = (2, 3, -2)$

$\vec{PR} = (0, 4, 2)$

$\triangle PQR = \frac{1}{2}\|\vec{PQ}\times\vec{PR}\|$

$= \frac{1}{2}\|(14, -4, 8)\|$

$= \sqrt{69}$

8. Explain geometrically why the equation

$$\langle 1,1,1\rangle \times X = \langle 1,0,0\rangle$$

has no solution. [§13.4 #62]

Let $X = \langle a, b, c\rangle$

$\Rightarrow \langle 1,1,1\rangle \times \langle a,b,c\rangle = \langle 1,0,0\rangle$

$\Rightarrow (*) \begin{cases} c-b=1 & -① \\ a-c=0 \Rightarrow c=a & -② \\ b-a=0 \Rightarrow b=a & -③ \end{cases}$

Using ② & ③ in ①,

$c-b = a-a = 0 \neq 1$

$\Rightarrow$ $(*)$ has no solution.

Hence, $\langle 1,1,1\rangle \times X = \langle 1,0,0\rangle$ has no solution.