Calculus Homework Assignment 5

Class: ____________

Student Number: ____________

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1. Find the equation of the plane which contains the lines $r_1(t) = \langle -t, 2t, -3t\rangle$ and $r_2(t) = \langle t, 2t, -6t\rangle$. [like §13.5 #25]

since the Plane contain $\vec{r_1}(t)$ and $\vec{r_2}(t)$
the direction vectors $\vec{v_1} = \langle -1, 2, -3\rangle$ and $\vec{v_2} = \langle 1, 2, -6\rangle$ of the lines lie in the plane.

$$\vec{n} = \vec{v_1} \times \vec{v_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 2 & -6 \end{vmatrix}\vec{i} - \begin{vmatrix} -1 & -3 \\ 1 & -6 \end{vmatrix}\vec{j} + \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix}\vec{k}$$

$= -6\vec{i} - 9\vec{j} - 4\vec{k} = \langle -6, -9, -4\rangle$

∵ $r_1(t) = \langle -t, 2t, -3t\rangle$ 都會在這平面裡

Choose $t = 1$ $(t=0$, 更好$)$ $\langle x_0, y_0, z_0\rangle = \langle -1, 2, -3\rangle$

$\vec{n} \cdot \langle x, y, z\rangle = \vec{n} \cdot \langle -1, 2, -3\rangle$

$\langle -6, -9, -4\rangle \langle x, y, z\rangle = \langle -6, -9, -4\rangle \langle -1, 2, -3\rangle$

$-6x - 9y - 4z = 6 - 18 + 12 = 0$

then $6x + 9y + 4z = 0$ ※

2. Find the intersection of the line

$$r(t) = \langle 2, -1, -1\rangle + t\,\langle 1, 2, -4\rangle$$

and the plane $2x + y = 3$. [§13.5 #31]

The parametric equations of the line are $x = 2 + t$, $y = -1 + 2t$, $z = -1 - 4t$

$2x + y = 3$

$\Rightarrow 2(2+t) + (-1+2t) = 3$

$\Rightarrow 4 + 2t - 1 + 2t = 3$

$\Rightarrow 4t = 0$

$\Rightarrow t = 0$

then $x = 2$, $y = -1$, $z = -1$

therefore the point P of intersection

$P = (2, -1, -1)$ ※

3. For which values of h is the intersection of the horizontal plane $x = h$ and the hyperboloid $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{4}\right)^2 - \left(\frac{z}{9}\right)^2 = 1$ empty? [§13.6 #25]

Let $x = h$

$\left(\frac{h}{2}\right)^2 - \left(\frac{y}{4}\right)^2 - \left(\frac{z}{9}\right)^2 = 1$

$\left(\frac{y}{4}\right)^2 + \left(\frac{z}{9}\right)^2 = \left(\frac{h}{2}\right)^2 - 1$

since $\left(\frac{y}{4}\right)^2 + \left(\frac{z}{9}\right)^2 \geq 0$

if $\left(\frac{h}{2}\right)^2 - 1 < 0$ the intersection is empty

$\left(\frac{h}{2}\right)^2 < 1 \Rightarrow h^2 < 4 \Rightarrow \cancel{h < |2|}$ $|h| < 2$

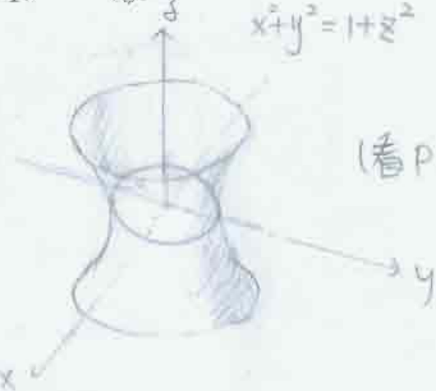
therefore the intersection of $x = h$ and $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{4}\right)^2 - \left(\frac{z}{9}\right)^2 = 1$ is empty when $\cancel{h < |2|}$ $|h| < 2$. ※

4. Sketch the given surface.

a. $x^2 + y^2 - z^2 = 1$

b. $z = \left(\frac{x}{4}\right)^2 - \left(\frac{y}{8}\right)^2$ [§13.6 #27, 30]

(a) $x^2 + y^2 = 1 + z^2$



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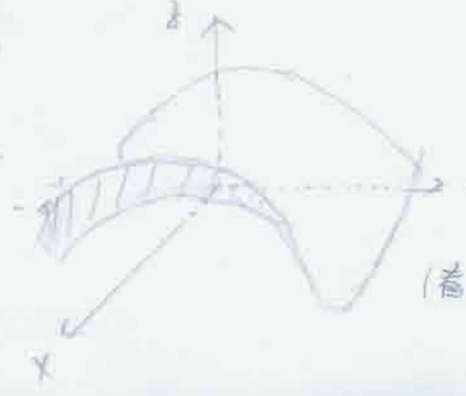
(b)



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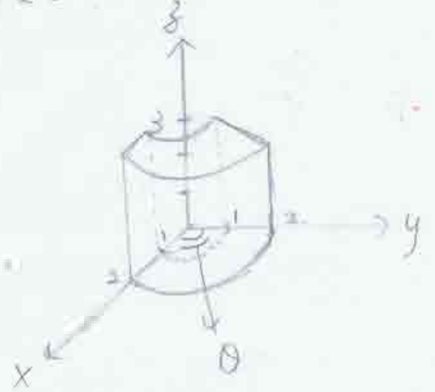
5. Sketch the set (described in cylindrical coordinates) where $1 \le r \le 2$, $0 \le \theta \le \frac{\pi}{2}$, and $0 \le z \le 3$. [like §13.7 #22]

$$\begin{cases} 1 \le r \le 2 \Rightarrow 1 \le \sqrt{x^2+y^2} \le 2 \\ 0 \le \theta \le \frac{\pi}{2} \\ 0 \le z \le 3 \end{cases}$$



7. Find a parametrization of the curve, the intersection of the plane $y = \frac{1}{2}$ with the sphere $x^2 + y^2 + z^2 = 1$. [§14.1 #31]

$y = \frac{1}{2}$, $x^2 + (\frac{1}{2})^2 + z^2 = 1$

$\Rightarrow x^2 + z^2 = \frac{3}{4} \Rightarrow \frac{x^2}{\frac{3}{4}} + \frac{z^2}{\frac{3}{4}} = 1$

let the parametrization

$x = \frac{\sqrt{3}}{2}\cos t$, $z = \frac{\sqrt{3}}{2}\sin t$.

and the point on the intersection formed $(\frac{\sqrt{3}}{2}\cos t, \frac{1}{2}, \frac{\sqrt{3}}{2}\sin t)$

$\vec{r}(t) = \langle \frac{\sqrt{3}}{2}\cos t, \frac{1}{2}, \frac{\sqrt{3}}{2}\sin t \rangle$ ※

6. Find an equation of the form $\rho = f(\theta, \phi)$ in spherical coordinates for the following surface.

$$z^2 = 3(x^2 + y^2)$$

[§13.7 #56] let $\begin{cases} x = \rho\cos\theta\sin\phi \\ y = \rho\sin\theta\sin\phi \\ z = \rho\cos\phi \end{cases}$

$z^2 = \rho^2\cos^2\phi = 3(\rho^2\cos^2\theta\sin^2\phi + \rho^2\sin^2\theta\sin^2\phi)$

$= 3\rho^2\sin^2\phi(\cos^2\theta + \sin^2\theta)$

$= 3\rho^2\sin^2\phi$

$\Rightarrow \rho^2\cos^2\phi = 3\rho^2\sin^2\phi$

$\Rightarrow \rho^2(\cos^2\phi - 3\sin^2\phi) = 0$

$\Rightarrow \rho = 0$ or $(\cos^2\phi - 3\sin^2\phi) = 0$

$\cos^2\phi = 3\sin^2\phi$

$\tan^2\phi = \frac{1}{3}$

$\tan\phi = \pm\frac{1}{\sqrt{3}}$

$\phi = \frac{\pi}{6}$ or $\frac{5}{6}\pi$

thus the equation is $\{\rho = 0$ or $\phi = \frac{\pi}{6}$ or $\frac{5}{6}\pi\}$ ※

8. Assume that two paths $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ intersect if there is a point P lying on both curves. We say that $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ collide if $\mathbf{r}_1(t_0) = \mathbf{r}_2(t_0)$ at some time t_0.
Determine whether $\mathbf{r}_1$ and $\mathbf{r}_2$ collide or intersect:
$\mathbf{r}_1(t) = \langle t, -t^3, 3t^2 - 2 \rangle$,
$\mathbf{r}_2(t) = \langle 4t + 6, 2t^2, 8 - t \rangle$. [like §14.1 #37]

if $\langle t, -t^3, 3t^2-2 \rangle = \langle 4t+6, 2t^2, 8-t \rangle$

$\begin{cases} t = 4t+6 \\ -t^3 = 2t^2 \\ 3t^2-2 = 8-t \end{cases} \Rightarrow \begin{cases} 3t+6 = 0 \\ t^2(2+t) = 0 \\ 3t^2+t-10 = 0 \end{cases} \Rightarrow \begin{cases} t = -2 \\ t = 0 \text{ or } -2 \\ t = -2, \frac{5}{3} \end{cases}$

therefore $t = -2$

there is a point $P = (-2, 8, 10)$ lying on both curves at the same $t = -2$.

$\vec{r}_1(t)$ and $\vec{r}_2(t)$ collide ※