Calculus Homework Assignment 7

Class: ______________________

Student Number: ______________________

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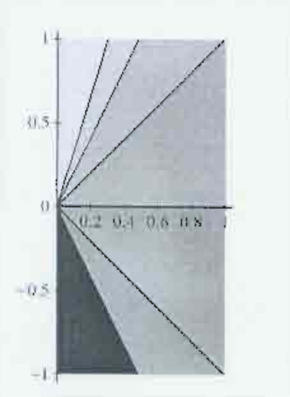
1. Draw a contour map of $f(x, y)$ with an appropriate contour interval, showing at least four level curves.

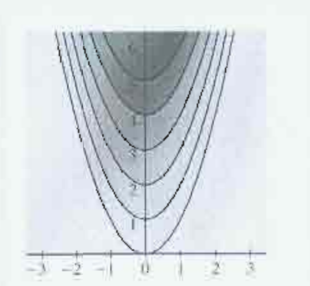
a. $f(x, y) = \dfrac{y}{x}$

b. $f(x, y) = x^2 - y$ [§15.1 #34, 40]

(a)



(b)



2. Refer to the Figure 25 on p.783. $f(s, t)$ denotes the density of seawater at salinity level S (parts per thousand) and temperature T (degrees Celsius).

a. Calculate the average ROC of density with respect to salinity from C to D.

b. At a fixed level of salinity, is seawater density an increasing or decreasing function of temperature? [§15.1 #47, 48]

(a) Average RoC from C to D is

$$\frac{\Delta d}{\Delta S} = \frac{0.0005}{0.8} = 0.000625 \ \text{kg}/\text{m}^3 \cdot \text{ppt}$$ ※

(b) Seawater density is a decreasing function of temperature ※

(Over Please)

3. Evaluate the limit or determine that the limit does not exist.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(\sin x)(\sin y)}{xy}$

b. $\displaystyle\lim_{(h,k)\to(2,0)} h^4 \frac{(2+k)^2 - 4}{k}$ [§15.2 #23, 27]

(a)

$\because \displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin x}{x}$ and $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin y}{y}$ exist

$\therefore \displaystyle\lim_{(x,y)\to(0,0)} \frac{(\sin x)(\sin y)}{xy} = \lim_{(x,y)\to(0,0)} \frac{\sin x}{x} \lim_{(x,y)\to(0,0)} \frac{\sin y}{y}$

$= 1 \cdot 1 = 1$

(b)

$\because \displaystyle\lim_{(h,k)\to(2,0)} h^4$ and $\displaystyle\lim_{(h,k)\to(2,0)} \frac{(2+k)^2-4}{k} = \lim_{(h,k)\to(2,0)} \frac{\cancel{k^2+4k+4-4}}{k}$ ※

$= \displaystyle\lim_{(h,k)\to(2,0)} (k+4)$ exist.

$\therefore \displaystyle\lim_{(h,k)\to(2,0)} h^4 \frac{(2+k)^2-4}{k}$

$= \displaystyle\lim_{h\to2} h^4 \lim_{k\to0} \frac{(2+k)^2-4}{k} = 2^4 \cdot 4 = 64$

※

4. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$ does not exist. [§15.2 #34]

Let $y = mx$

$\Rightarrow f(x, y) = \dfrac{y^2}{x^2+y^2} = \dfrac{(mx)^2}{x^2+(mx)^2} = \dfrac{m^2}{1+m^2}$

$\because \displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ depends on m

$\therefore \displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist

※

5. Compute the derivative indicated.

$$g(x,y) = xye^{-y}, \qquad g_{yyx}(1,0)$$

[like §15.3 #56]

$\because g_y(x,y) = \frac{\partial}{\partial y} g(x,y) = \frac{\partial}{\partial y}(xye^{-y}) = x\frac{\partial}{\partial y}(ye^{-y})$

$= x(1\cdot e^{-y} + y\cdot e^{-y}\cdot(-1)) = xe^{-y}(1-y)$

$g_{yy}(x,y) = \frac{\partial}{\partial y}(g_y(x,y)) = \frac{\partial}{\partial y}[xe^{-y}(1-y)]$

$= x\frac{\partial}{\partial y}(e^{-y}(1-y)) = x(-e^{-y}(1-y)+e^{-y}(-1))$

$= xe^{-y}(y-2)$

$g_{yyx}(x,y) = \frac{\partial}{\partial x}[g_{yy}(x,y)] = \frac{\partial}{\partial x}[xe^{-y}(y-2)]$

$= e^{-y}(y-2)\frac{\partial}{\partial x}(x) = e^{-y}(y-2)$

$\therefore g_{yyx}(1,0) = e^{-0}(0-2) = 1\cdot(-2) = -2$

※

6. Let

$$f(x,y,u,v) = \frac{x^2+e^y v}{3y^2+\ln(2+u^2)}$$

What is the fast way to show that

$f_{uvxyvu}(x,y,u,v) = 0$ for all (x,y,u,v)?

[§15.3 #62]

$\because f_v(x,y,u,v) = \frac{\partial}{\partial v}\left(\frac{x^2}{3y^2+\ln(2+u^2)}\right) + \frac{\partial}{\partial v}\left(\frac{e^y}{3y^2+\ln(2+u^2)}v\right)$

$= 0 + \frac{e^y}{3y^2+\ln(2+u^2)} = \frac{e^y}{3y^2+\ln(2+u^2)}$

and $f_{vx}(x,y,u,v) = \frac{\partial}{\partial x}\left(\frac{e^y}{3y^2+\ln(2+u^2)}\right) = 0$

$\therefore f_{uvxyvu}(x,y,u,v) = 0$

※

7. Find the points on the graph of $z = 3x^2 - 4y^2$ at which the vector $\mathbf{n} = \langle 3,2,2\rangle$ is normal to the tangent plane. [§15.4 #19]

Let $P(a,b,f(a,b))$

$\Rightarrow$ the tangent plane at P on $z=f(x,y)$ is

$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\Rightarrow f_x(a,b)(x-a) + f_y(a,b)(y-b) - z + f(a,b) = 0$

Let $v = \langle f_x(a,b), f_y(a,b), -1\rangle$

$f_x(x,y) = 6x \quad f_x(a,b) = 6a$

$f_y(x,y) = -8y \quad f_y(a,b) = -8b$

$\Rightarrow v = \langle 6a, -8b, -1\rangle$ and $v \parallel n$

where $n = \langle 3,2,2\rangle$

$\Rightarrow \frac{6a}{3} = \frac{-8b}{2} = \frac{-1}{2}$

$\Rightarrow a = -\frac{1}{4} \quad b = \frac{1}{8}$

$\Rightarrow z = 3(-\frac{1}{4})^2 - 4(\frac{1}{8})^2 = \frac{1}{8}$

$\Rightarrow P(-\frac{1}{4}, \frac{1}{8}, \frac{1}{8})$

※

8. Use the linear approximation to estimate the value $(2.01)^3(1.02)^2$. Compare with the value given by a calculator, 8.4487. [§15.4 #25]

(1) Let $f(x,y) = x^3y^2$

$\because f(2+h, 1+k) \approx f(2,1) + f_x(2,1)h + f_y(2,1)k$

and

$f(x,y) = x^3y^2, \ f(2,1) = 2^3\cdot 1^2 = 8$

$f_x(x,y) = 3x^2y^2, \ f_x(2,1) = 3\cdot 2^2\cdot 1^2 = 12$

$f_y(x,y) = 2x^3y, \ f_y(2,1) = 2\cdot 2^3\cdot 1 = 16$

$h = 0.01 \quad k = 0.02$

$\therefore (2.01)^3(1.02)^2 = (2+0.01)^3(1+0.02)^2$

$\approx f(2,1) + f_x(2,1)\cdot 0.01 + f_y(2,1)\cdot 0.02$

$= 8 + 12\cdot 0.01 + 16\cdot 0.02 = 8.44$ ※

(2)

$\because$ The value given by a calculator is 8.4487

$\therefore$ error $\approx \frac{8.4487-8.44}{8.4487}\times 100\% \approx 0.103\%$

※