Ordinary Differential Equations Final Examination

June 30, 2015

Choose five problems from 1-6.

1. Consider the following initial value problem

$$\begin{cases} x' = x(1 - x - y) \\ y' = y(x - \lambda), & 0 < \lambda < 1 \\ x(0) > 0, & y(0) > 0. \end{cases}$$

Disscuss the structure of all solutions.

2. Consider the Van Der Pal Equation

$$\begin{cases} \ddot{x} + \epsilon (x^2 - 1)\dot{x} + x = 0, \\ x(0) = x_1, \ \dot{x}(0) = x_2. \end{cases}$$

Disscuss the structure of all solutions.

3. Consider the following initial value problem

$$\begin{cases} x'(t) = r_1 x(t) \left[1 - \frac{x(t)}{\kappa_1} \right] - \alpha x(t) y(t) \text{ on } [0, \infty), \\ y'(t) = r_2 y(t) \left[1 - \frac{y(t)}{\kappa_2} \right] - \beta x(t) y(t) \text{ on } [0, \infty), \\ x(0) = x_0 > 0, \ y(0) = y_0 > 0, \end{cases}$$

where α , β , r_1 , r_2 , κ_1 and κ_2 are positive constants. Disscuss the structure of all solutions.

4. Consider the following initial value problem

$$\begin{cases} x'(t) = [A - By(t)]x(t) \text{ on } [0, \infty), \\ y'(t) = [Cx(t) - D]y(t) \text{ on } [0, \infty), \\ x(0) = x_0 > 0, \ y(0) = y_0 > 0, \end{cases}$$

where A, B, C and D are positive constants. Let X(t) = (x(t), y(t)) be a solution of (II) on $[0, \infty)$. Disscuss the structure of all solutions.

5. Consider the Duffing equation

$$\ddot{x} + \beta \dot{x} + (ax^3 + bx) = 0,$$

where $a > 0, b \in R, \beta \in R$. Disscuss the structure of all solutions.

6. Consider the Van-Der Pol equation

$$\ddot{x} + x + \varepsilon (x^2 - 1)\dot{x} = 0,$$

Disscuss the structure of all solutions.