Ordinary Differential Equations Mid-term Test

May 5, 2015

1. Consider the following initial value problem

$$\begin{cases} x = x(1-x-y) \\ y = y(x-\lambda), \quad 0<\lambda<1 \\ x(0)>0,\ y(0)>0. \end{cases}$$

Prove that $\lim_{t\to\infty}$(x(t),y(t))=(λ,1-λ).

2. Consider the Van Der Pal Equation

$$\begin{cases} \ddot{x}+\epsilon(x^2-1)\dot{x}+x, \\ x(0)=x_1,\ \dot{x}(0)=x_2. \end{cases}$$

Prove that for any $\epsilon<0$, then every solution x(t) starts in the circle $(x_1,x_2)\in\{(x_1,x_2)|x_1^2+y_1^2<\epsilon\}$ approaches to zero as $t\to\infty$.

3. Consider the following initial value problem

$$(*)\ \ddot{x}+f(x)\dot{x}+h(x)=0.$$

Prove : If1.$xh(x)>0,\ for\ any\ x\neq 0$
2.$f(x)>0,\ for\ any\ x\neq 0$
3.$H(x)=\int_0^x h(t)\,\mathrm{d}t\to\infty as\ |x|\to\infty$,
then every soiution x(t) of(*) satisfies $\lim_{t\to\infty}(x(t),\dot{x}(t))=(0,0)$.

4. Consider the following initial value problem

$$\begin{cases} x'=rx(1-\dfrac{x}{k})-\dfrac{mxy}{a+x} \\ y'=y(\dfrac{mx}{a+x}-D) \\ x(0)>0,\ y(0)>0. \end{cases}$$

Prove that if $0<\frac{k-a}{2}<\frac{aD}{m-D}<k$, then $\lim_{t\to\infty}$(x(t),y(t))=(x^*,y^*),
$y^*=f(x^*),\ f(x)=\frac{r(a+x)}{m}(1-\frac{x}{k})$.