(8選5題作答)

P1

Final exam of ordinary differential equation

2018.1.10

1. An equation of the form

$$t^2\frac{d^2y}{dt^2}+\alpha t\frac{dy}{dt}+\beta y=0,\quad t>0 \tag{1}$$

where α and β are real constants, is called an Euler Equation.

(a) Let $x=lnt$ and calculate $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ in terms of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Use the results of part (a) to transform equation (1) into

$$\frac{d^2y}{dx^2}+(\alpha-1)\frac{dy}{dx}+\beta y=0 \tag{2}$$

Observe that differential equation (2) has constant coefficients.

(c) Use the method which is introduced in (a)(b) to solve the given equation for $t>0$

$$t^2y''-3ty'-12y=0 \tag{3}$$

2. If a, b and c are positive constants, show that all solutions of

$$ay''(t)+by'(t)+cy(t)=0$$

approach zero as $t\to\infty$

3. (a) Show that if $(N_x-M_y)/(xM-yN)=R$, where R depend on the quantity xy only, then the differential equation

$$M+Ny'=0$$

has as integrating factor of the form $\mu(xy)$. Find a general formula for this integrating factor.

(b) Find an integrating factor and solve the given equation

$$[4(x^3/y^2)+(3/y)]+[3(x/y^2)+2y]y'=0$$

4. 試求出下列二階 ODE 的一特殊解

① $-y'' + 3y' + 4y = -3e^{2t}$

② $-y'' + 3y' + 4y = -2\sin t$

③ $-y'' + 3y' + 4y = 8e^{t}\cos 2t$

④ $y'' - 3y' - 4y = 3e^{2t} + 2\sin t$

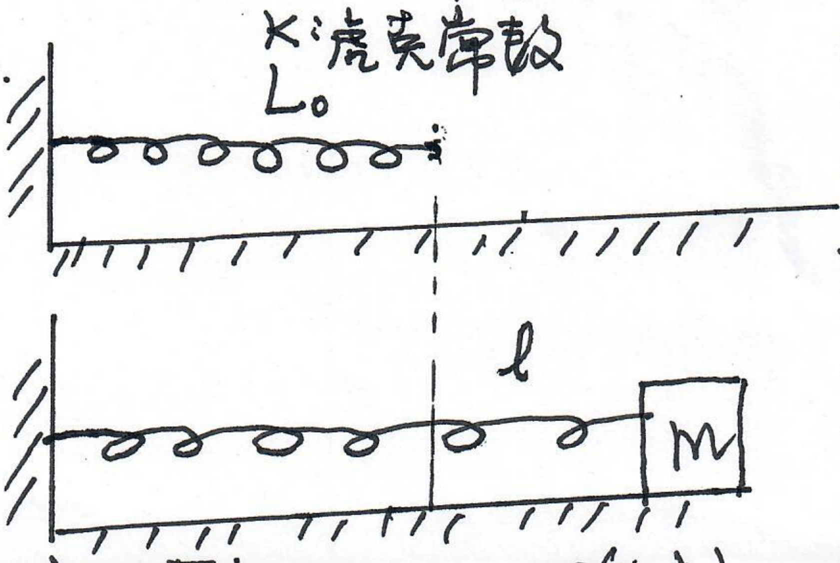
5 ① 令 a, b, c 是 3 個實係數
試依 a, b, c 的條件，求下列 ODE
的一般解.

$$ay'' + by' + cy = 0$$

② 試解出下列 ODE 的唯一解.

$$\begin{cases} y'' + y' + 9.25y = 0 \\ y(0) = 1,\ y'(0) = 2. \end{cases}$$

6.



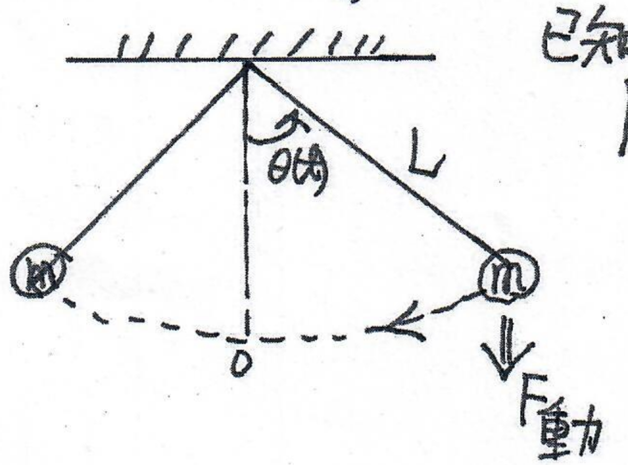
如左圖：一彈簧原長為 L_0 單位，虎克常數 K
若在尾部掛上一質量為 m 的物體後拉長 l 單位後放
試導出運動方程式

假設 $F_{摩擦力} \propto v$(速度)

試証 $\lim v(t) = 0$

7. 探討下列問題的微分方程 models

(A) 同一單擺分別置於地球表面及月球表面作來回運動模型.(不計摩擦力及地球上的空氣阻力) 當此單擺是作微小角度擺動時,試比較其週期性.(已知月球質量大約 $=\frac{1}{81}$ 地球質量)



已知:
$|\theta(t)| < \theta_0 << 1 \quad \forall t$

$(F_{重力})_{地} = m g_{地球}$

$(F_{重力})_{月} = m g_{月球}$

(B) A Falling Object:

f_g: the drag force

Assume $f_d \propto v$ (Case 1)

$f_d \propto v^2$ (Case 2)

8 Solve the following ODE respectively. Find $\lim_{t\to\infty} y(t)$.

(1) $\frac{dy}{dt} = -r\left(1-\frac{y}{T}\right)y$, $y(0)=y_0>0$, r, T: positive constant.

(2) $\frac{dy}{dt} = -r\left(\frac{y}{T}-1\right)\left(\frac{y}{K}-1\right)y$, $r>0$, $0<T<K$: constants

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where α and β are real constants, is called an Euler Equation.

(a) Let $x = lnt$ and calculate $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ in terms of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

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$$\frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0 \tag{2}$$

Observe that differential equation (2) has constant coefficients.

(c) Use the method which is introduced in (a)(b) to solve the given equation for $t > 0$

$$t^2y'' + 3ty' - 8y = 0 \tag{3}$$

2. If a, b and c are positive constants, show that all solutions of

$$ay''(t) + by'(t) + cy(t) = 0$$

approach zero as $t \longrightarrow \infty$

3. Solve the following initival value problems repectively.

(1) $\begin{cases} (16-t^2)y' + 2ty = 3t^2 \\ y(1) = -5 \end{cases}$

(2) $\begin{cases} 2y' + ty = 2 \\ y(0) = 1 \end{cases}$

(3) $\dfrac{dy}{dx} = \dfrac{4x - x^3}{4 + y^3}, \quad y(0) = 1$

(4) $\begin{cases} (3xy + y^2) + (x^2 + xy)y' = 0 \\ y(1) = 1 \end{cases}$

4. 試求出下列二階ODE的一特殊解

① $-2y''+6y'+8y=+6e^{2t}$

② $-2y''+6y'+8y=+4\sin t$

③ $-2y''+6y'+8y=16e^{t}\cos 2t$

④ $-2y''+6y'+8y=6e^{2t}+4\sin t$

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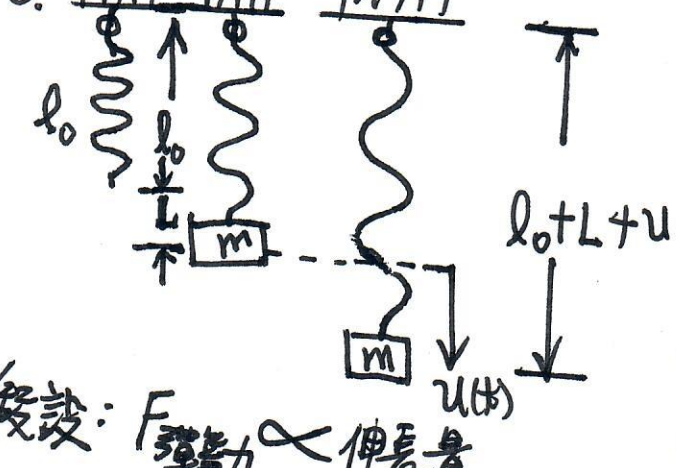
試依 a, b, c 的條件, 求下列 ODE 的一般解.

$$ay''+by'+cy=0$$

② 試解出下列 ODE 的唯一解.

$$\begin{cases} y''+y'+\dfrac{37}{4}y=0 \\ y(0)=2 \quad y'(0)=1 \end{cases}$$

6.



如左圖: 一彈簧原長為 l_0 單位 虎克常數為 k. 若在尾部垂直掛上一質量為 m 單位物体 至靜止時彈簧伸長 L 單位 今假設此物体的重量在此彈簧的彈性限度內. 試探討在原彈簧瞬間掛上

假設: $F_{彈力} \propto$ 伸長量.

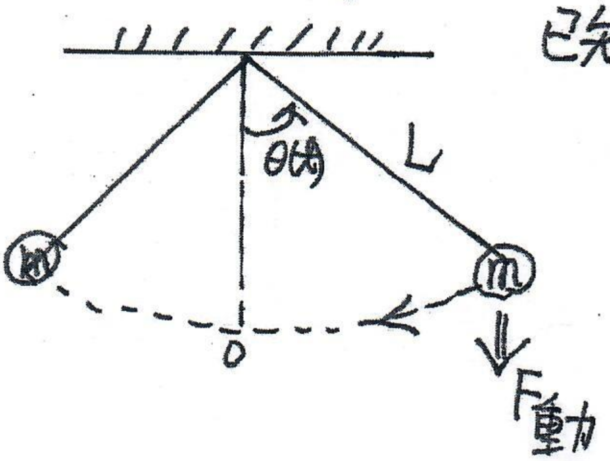
$F_{阻力} \propto$ 速度 比例常數

7. Consider the (IVP) $\begin{cases} \frac{dy}{dt} = 3 - 2t - 0.5y \\ y(0) = 1. \end{cases}$

Use Euler's method: $y_{n+1} = y_n + f(t_n, y_n)(t_{n+1} - t_n)$, $n = 0, 1, 2, \cdots$ with step size $h = 0.2$ to find approximation of the solution at $t = 0.2, 0.4, 0.6, 0.8$.

8. 探討下列問題的微分方程 models (P3)

(A) 同一單擺分別置於地球表面及月球表面作來回運動模型.(不計摩擦力及地球上的空氣阻力)當此單擺是作微小角度擺動時,試比較其週期性.(已知月球質量大約$=\frac{1}{81}$地球質量.)



已知:

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$(F_{重力})_{地} = m g_{地球}$

$(F_{重力})_{月} = m g_{月球}$

(B) A Falling Object:

Assume $f_d \propto v$ (Case 1)

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