ORDINARY DIFFERENTIAL EQUATIONS MIDTERM EXAM

Please choose three from all questions to answer.

1. State and prove the local existence of solutions of the following initial value problem
$$\begin{cases} \dfrac{dx'}{dt} = f(t,x) \\ x(t_0) = x_0 \end{cases},$$
where $f : D \subseteq R \times R^n \to R^n$

2. State and prove the uniqueness of solutions of the following initial value problem
$$\begin{cases} \dfrac{dx'}{dt} = f(t,x) \\ x(t_0) = x_0 \end{cases},$$
where $f : D \subseteq R \times R^n \to R^n$

3. Let $y(t,\alpha)$ be the solution of the following ODE
$$\begin{cases} y'(t) = \beta y(t)(K_0 - y(t)) \\ y(0) = \alpha > 0 \end{cases},$$
where β and K_0 are two positive constants. Prove $lim_{t\to\infty} y(t,\alpha) = K_0$ for all $\alpha > 0$

4. Show that if $g \in C^1(R)$ and $f \in C(R)$ then the solution of I.V.P.
$$y'' + f(y)y' + g(y) = 0,\ y(t_0) = A,\ y'(t_0) = B$$
exists locally, is unique and can be continued so long as y and y' remain bounded.

5. Let $x(t,\alpha)$ be the solution of
$$\begin{cases} x' = f(t,x) \\ x(0) = \alpha \end{cases},$$
where f, $\frac{\partial f}{\partial x}$ are continuous from $R \times R \to R$
Prove that if $\frac{\partial f}{\partial x}(t,x) \geq 0$ for all (t,x), then $x(t,\alpha_1) > x(t,\alpha_2)$ for all $t > 0$ and for all $\alpha_1 > \alpha_2$